# Fast neutron spectrum measurement with threshold detectors 

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FAST NEUTRON SPECTRUM MEASUREMENT WLIH THRESHDLD DETECTIRS.

IUWA STATE UNIVERSITY, PH.D., 1979

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# Fast neutron spectrum measurement with threshold detectors 

 byMohammad Moghari

A Dissertation Submitted to the Graduate Faculty in Partial Fulfillment of The Requirements for the Degree of DOCTOR OF PHILOSOPHY<br>Major: Nuclear Engineering

## Approved:

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## INTRODUCTION

The knowledge of the neutron flux and energy spectrum of a given reactor facility is essential for the evaluation of experimental results involving irradiation of materials. Furthermore, the interpretation of experiments dealing with such disciplines as radiobiology, radiochemistry, as well as the study of radiation damage produced by fast neutrons require an accurate knowledge of neutron flux and spectrum. For instance, the phenomenon of neutron-produced increases in the ductile-to-brittle transition temperature of ferritic steels has been studied extensively. The experimental studies conducted have permitted the formulation of a theory for the increase in transition temperature as a function of neutron fluence, typically reported as neutrons per square centimeter with energies greater than 0.1 MeV (or occasionally greater than 1 MeV .

The criterion or assumption has traditionally been made that only when neutrons have energies above a specified threshold will damage result and that neutron with energies above this threshold are equally effective in causing damage. In nearly all studies involving radiation damage it is assumed that neutrons of energies greater than 0.1 MeV all contribute equally to the damage process. Although this assumption has served successfully for many types of studies, there have been instances where it has failed to yield acceptable corre-
lation between fluence and neutron-induced embrittlement. This lack of agreement has been traced to the energy spectrum of the neutron radiation environment which, in most cases, is markedly different from the assumed step function.

Generally in the field of radiation effects on materials, changes in physical properties produced by transmutation reactions require an accurate knowledge of neutron spectrum beiore theory and experiment can be adequately correlated. Furthermore, the present incomplete understanding of the dama.ge producing mechanism hampers attempts to correlate neutron fluence with the neutron-induced damage in the specimen. Consequentiy, in order to analyze and compare results obtained by irradiation studies - and especially - in different reactor facilities, it is necessary to have an accurate knowledge of the energy dependence of the neutron flux, i.e. the spectrum and the neutron fluence.

The best way of measuring reactor neutron spectra, with energies ranging typically from approximately $10^{-10} \mathrm{MeV}$ to about 15 MeV , is to irradiate activation foils whose energy dependent cross sections for a particular reaction are known. The induced activities are then measured and used to find, with one of the spectrum uniolding techniques, a neutron spectrum capable of explaining the observed activities.

The objectives of this investigation were a) to select a system of activation detectors that will enable the experi-
menter to determine the neutron flux and spectum at an enexgies, and b) to measure the neutron spectra in two Ames Laboratory Research Reactor (ALRR) radiation facilities.

Given the higher damasing capability of the fast neutrons, more emphasis is placed upon the determination of the spectrum a.t energies greater than 100 KeV . Hence, the activation foils used in this work were in most cases selected to be threshold detectors.

The neutron induced activities in the irradiated foils were analyzed and measured with the aid of a Ge(Li) detector and associated counting system. In order to determine the spectrum from the measured activities, use was made of a modified SAND-II (Spectrum Anelysis by Neutron Detectors) computer code. SAFD-II is based on an iterative unfolding technique which provides a best fit neutron spectrum for a given set of experimentally determined foil activites. The determination of the errors involved in this work was carried out using a Monte Carlo error analysis code. The code takes into account experimental errors in collecting the experimental data as well as errors related to the SAND-II unfolding technicue.

## I. IITERATURE SEARCH

Various techniques are available for the measurement of neutron spectrum; these include nuclear emulsions, proton recoil spectrometry, time-Of-flight techniques, and neutron activation. However, when measurements are to be made inside the reactor and the spectrum runs from thermal to fast neutron energies, only neutron activation turns out to be practical.

Neutron activation was originally used in 1936 by von Heresy and Levi (1) to detect the presence of dysprosium and europium in rare earth mixtures by means of their measured activities. Neutron activation for the measurement of neutron spectra was considered in 1951. Cohen (2) was the first to consider the use of activation detectors for the measurement of neutron energies above the thermal energy region. He suggested the use of ( $n, p$ ) and ( $n, 2 n$ ) threshold detectors for the energy range between 1 and 25 MeV . In 1956 Hurst (3) extended the energy coverage of threshold detectors to the KeV region by introducing such fission detectors as ${ }^{239} \mathrm{Pu},{ }^{238} \mathrm{U}_{\mathrm{U}}$, and ${ }^{237}$ Np.

Hurst used boron shielding to give the ${ }^{239} \mathrm{Pu}$ ( $n$,fission) reaction an effective threshold enerof of approximately 1 KeV . ${ }^{238} \mathrm{U}$ ( $n$, iission) and ${ }^{237}$ inp ( $n$, îission) reactions with threshold energies oi about 0.6 MeV and 1.5 HeV respectively were also used.

Details for boron sinelding geometry irradiation consid-
erations, and celibrations procedures for the above reactions and for the ${ }^{32} S(n, p){ }^{32}$ p reaction were given.

In 1958, Reinhardt and Davis (4) presented a number of improvements in the use of fission detectors. These included techniques for sample preparation and fission reaction standardization with the sulfur threshold reaction. Following Reinhardt's paper, Hurst and Ritchie (5) described a new calibration procedure and introduced the concept of the "equivalent foil technique" for calibrating counting sysīems used for counting the gamma radiation from neutron activated fission detectors.

Within recent years many publications have appeared which form the basis for many of the techniques, both experimental and mathemaiical, currently in use for neutron spectrum measurement. Wechsler and Trice (6) described general procedures for neutron spectrum reasurement in connection with radiation effects on materials. Stilier (7) used ${ }^{31} \mathrm{P}(\mathrm{n}, \mathrm{p})^{31} \mathrm{Si}$;
 ${ }^{2.4} 4_{\mathrm{Mg}}(\mathrm{n}, \mathrm{n})^{24} \mathrm{Na} ;{ }^{27} \mathrm{Al}(\mathrm{n}, \alpha)^{24} \mathrm{Na} ;{ }^{63} \mathrm{Cu}(\mathrm{n}, 2 \mathrm{n})^{62} \mathrm{Cu}$; ${ }^{28}$ Si $(n, \underline{n})^{28}$ 思; threshold reactions for the measurement of fast neutron spectra. He adopted Grundl and Usner's (E) method of effective threshold for spectrum unfolding. Levine ei 21. (9) presented the procedure used for neutron spectrum measurement in the Northrop Triga Reactor.

In 1965 Ziju ( 10,11 ) presented a comprenensive review of
activation methods for the determination of neutron spectra. In addition to the basic concepts, Zijp presented details of threshold reactions of interest and reported some important methods for the determination of a neutron spectrum from the response integral of the activation detectors. McElroy et al. (12) reported a manual iterative unfolding method to obtain neutron spectra from the results of foil activation. Their enalytical studies showed that integral neutron flux might be obtained from foil data rith accuracies to within $\pm 10 \%$ to $\pm 30 \%$ at any point over the energy range from $4 \times 10^{-7}$ to 18 MeV , if the activation cross section data and measured activation were accurate to $\pm 10 \%$.

Later, McElroy et al. (13) used a fully-automated computer code to perform spectral analysis by a substantially improved iterative method. This multiple foil activation iterative method, called SAND-II (14) has been used to experimentally determine the neutron spectra in verious types of neutron environment. Comparisons with reactor physics calculations and spectrometer measurements indicated that the method should give results with accuracies between $\div 10 \%$ and $\pm 30 \%$, depending on experimental conditions.

Gruncl (15) employed eight activation detectors to compare the thermal-neutron-induced fission-neutron spectra of 235 U , 233 U , and ${ }^{239} \mathrm{Pu}$. The activation reactions were as follows: ${ }^{235}$

${ }^{56} \mathrm{Fe}(\mathrm{n}, \mathrm{p})^{56} \mathrm{Mn}$; ${ }^{27} \mathrm{Al}(\mathrm{n}, \alpha)^{24} \mathrm{Na}$; and ${ }^{63} \mathrm{Cu}(\mathrm{n}, 2 \mathrm{n})^{62} \mathrm{Cu}$. The eight detectors provided a good sampling of the spectra in wide, but distinct, energy intervals. The results were generally in good agreement with other methods of spectrum determination such as time-of-flight and nuclear emulsion. Dodds (16) utilized seven activation reactions to measure the fast neutron spectrum in the permanent beryllium reflector of the High Flux Isotope Reactor. The reactions employed were: ${ }^{27} \mathrm{Al}(\mathrm{n}, \alpha)^{24} \mathrm{Na} ;{ }^{56} \mathrm{Fe}(\mathrm{n}, \mathrm{p}){ }^{56} \mathrm{Mn} ;{ }^{54} \mathrm{Fe}(\mathrm{n}, \mathrm{p}){ }^{54} \mathrm{Mn} ;{ }^{238} \mathrm{U}(\mathrm{n}, \mathrm{f}){ }^{140} \mathrm{Ba}$; $5^{58_{\mathrm{Ni}}(\mathrm{n}, \mathrm{p})}{ }^{58} \mathrm{Co} ;{ }^{24} \mathrm{Mg}(\mathrm{n}, \mathrm{p}){ }^{24} \mathrm{Na}$; and ${ }^{32} \mathrm{~S}(\mathrm{n}, \mathrm{p}){ }^{32} \mathrm{P}$. The author presents a good description of the materials and apparatus associated with the irradiations, counting equipment, and an excellent analysis of the Relative Deviation Minimization Method (RDMM) (17), the unfolding technique employed to obtain the solution.

McElroy and Kellog (18) reported the progress made in achieving high-accuracy measurements of fission rates, fuel burnup, and neutron exposure for material property changes through a coordinated interlaboratory effort. In addition to such subjects as foil set selection, neutron self-shielding corrections, and perturbation effects, the paper reports a number of improvements that have been made in the SAND-II algorithm. The improved procedure produces a smooth realistic appearing curve by reducing the amount of artificial structure originally generated by the iterative method of solution.

Greenwood et al. (19) reported an interlaboratory compar-
ison of the reaction rates measured by five grouns. Nonissile activation foils were irradiated in the Coupled Fast Reactivity lieasurements Facility and gamma counted. Each group used an independently calibrated Ge(Ii) detector, and in some cases a NaI (TI) detector as well. The reaction rate values were all in good agreement, generally consistent to within $\pm 2 \%$ with respect to each other. Based upon this report, it was concluded that for nonfissile foils, where the nuclear data are adequate, reaction rates can be determined with an accuracy of approximately $\div 2 \%$.

As foil activation techniques gained increasing use in the area of spectrum measurement, many techniques were developed to calculate differential neutron flux from foil activation daia. A review of these techniques is presented in section II.

## II. THEORETICAL DEVELOPMENT

## A. Spectrum Unfolding Techniques

The foil activation method for measuring neutron energy spectra involves the irradiation and activation of a set of selected foils followed by the subsequent determination of their saturated activities. Let $m$ and $n$ respectively be the number of target atoms and the reaction product atoms at time $t$ and $m_{0}$ be the initial number of target atoms (at $t=0$ ). The specific saturated activity (or response integral) Asp of a given foil is
$A_{s p}=\frac{A}{m_{0}}=\int_{0}^{\infty} \sigma(E) \varphi(E) d E$
where $\sigma(E)$ is the energy dependent microscopic cross section and $\phi(E)$ is the steady state differential flux*. If $N$ activation foils are irradiated, there are $\mathbb{N}$ response integrals defined by Equation (1) which could be solved for $\Phi(E)$.

Generally the number of energy groups of interest, $M$, is larger than the number of reactions used, and therefore there are $N$ equations with $M>N$ unknowns. To reach a solution, some assumptions have to be made as to the analytical form of $p(\mathbb{E})$. Based upon the assumptions chosen, the methods to solve Equation (1) for $\varphi(\mathbb{E})$ can be divided into three mair groups:

[^1]1. Mathematical methods
2. Perturbation methods
3. Weighting methods

As will be seen in the following pages, the above classification does not provide a sharp distinction among the various unfolding methods. This is because there are cases which feature an overlap of methods.

1. Mathematical methods

The assumptions concerning the analytical form of $\varphi(E)$ are purely mathematical. The methods which fall into this category include a) Flux Integral b) Step Function Approximation c) Polygonal d) Simple Polynonials e) Expansion in Orthonormal Combinations of Cross Section Curves f) Expansion in Orthonormal Combinations of Simple Polynomials and g) Successive Exponentials.
a) Flux integral method In this method; as discussed by Hughes (20), the cross sections for the threshold reactions used are assumed to be step functions. The step functions are characterized by their $\sigma_{e f f}$ and Eeff values. Therefore, the response integral of a detector can be expressed as
$A_{S D}=\int_{0}^{\infty} \sigma(E) \varphi(E) d E=\sigma_{e f f} \int_{E I f}^{\infty} \varphi(E) d E$
Values oi Eeff for each threshold reaction employed are obtained by using an assumed fission spectrum for calibration as explained below.

The following is an example of how an unknown spectrum is obtained using this method. Suppose there are two different reactions characterized by $\sigma_{\text {eff }}, E_{\text {effi }}$ and $\sigma_{\text {eff }}$, $E_{\text {eff2 }}$. The experimentally determined response integrals are given by
$A_{s p 1}=\int_{0}^{\infty} \phi(E) \sigma_{1}(E) d E=\sigma_{\text {eff }} \int_{E_{e f f}}^{E_{\max }} \varphi(E) d E=\sigma_{e f f_{1}} \Phi_{1}(1-A)$
$A_{S D 2}=\int_{0}^{\infty} \varphi(E) \sigma_{2}(E) d E=\sigma_{e f \hat{I}_{2}} \int_{E_{e f f 2}}^{E_{\max }} \varphi(E) d E=\tilde{\sigma}_{e f f_{2}} \Phi_{2}(1-B)$
In Equations ( $1-A$ ) and ( $1-B$ ), $\mathcal{\sigma}_{e f f_{1}}$ and $\sigma_{e f f 2}$ represent an average value of the cross sections $\sigma_{1}(\mathbb{E})$ and $\sigma_{2}(E)$. If $A_{s p 1}$ and $A_{s p 2}$ are measured and values of $\sigma_{\text {eff }}$ and Eeff are selected, then the neutron flux in the energy interval $\mathrm{E}_{\text {eff2 }}-\mathrm{E}_{\text {effif }}$ can be calculated

$$
\Phi_{12}=\Phi_{1}-\Phi_{2}=\int_{E_{e f f_{1}}}^{E_{e f \tilde{f}_{2}}} \varphi(\mathbb{E}) d E=\frac{A_{S p 1}}{\sigma_{e f \tilde{f}_{1}}}-\frac{A_{s p 2}}{\sigma_{e f f_{2}}}
$$

Extending the above procedure to a system of $N$ reactions, the flux integrals could be evaluated for N-1 intervals, Yel= ues for $\sigma_{e f f_{i}}$ and $E_{e f f_{i}}$ are selected as follows:

- $\sigma_{\text {efif }}$ is selected as the best average value of $G_{i}(\mathbb{E})$ over the energy interval of interest $\left(\Xi_{E f f_{i}}-E_{m a x}\right)$, and equal to zero elsewhere.
- Once the value of $\theta$ is chosen, a fission spectrum
(e.g. Cramberg or Watt) is selected to replace $\varphi(E)$ in Equation (1-A) part A. This part of Equation (1-A) can now be integrated to yield a value for Eeffi and finally the flux can be evaluated.

This is the simplest method; however, it provides no information for energies below the lowest threshold energy. Furthermore, its use is limited to cases where the actual spectrum is a close approximation to the fission spectrum.
b) Step function approximation In this method as outlined by Delattre, the total energy renge of the fast neutrons is divided into $N$ energy intervals; $N$ being the number of threshold reactions used. The flux is assumed constant in each energy region. The energy range starts with $E_{1}$ as the lowest energy threshold and ends with $E_{N+1}$ as the highest energy threshold (21, 22).

For the ith reaction the response integral is expressed as

$$
\begin{align*}
A_{s p i}= & \int_{E_{1}}^{E_{N+1}} \sigma i(E) \varphi(E) d E=\phi_{1} \frac{\int_{E_{1}}^{E_{2}} \sigma E_{1}}{E_{2}(E) d E}+ \\
& +\varphi_{2} \frac{E_{E_{2}} \sigma i(E) d E}{E_{3}-E_{2}} \quad \int_{E_{N}}^{E_{N+i}} \sigma i(E) d E \tag{2}
\end{align*}
$$

Since $\sigma i(E)$ are considered to be known, $\varphi_{1}, \varphi_{2}$, $\varphi_{\mathbb{N}+1} \mathrm{can}$ be solved by solving $\mathbb{N}$ linear equations with $\mathbb{N}$ unknowns as defined by Equation (2). The solution provides a step function approximation to the actual spectrum. The set of Equations (2) can be written in a matrix notation:

$$
\begin{equation*}
[A]=[b][\phi] \tag{2-A}
\end{equation*}
$$

where

$$
\begin{aligned}
& {[A]=\left|\begin{array}{c}
A_{1} \\
A_{2} \\
\vdots \\
A_{N}
\end{array}\right| \quad A_{i}=A_{\text {spi }}} \\
& {[\varphi]=\left|\begin{array}{l}
\varphi_{1} \\
\varphi_{2} \\
\vdots \\
\varphi_{N}
\end{array}\right|}
\end{aligned}
$$

and $b_{i j}=$ general term of $[b]$

$$
b_{i j}=\frac{1}{E_{j+1}-E_{j}} \int_{E_{j}}^{E_{j+1}} \sigma i(E) d E \quad \text { with } \begin{aligned}
& i=1, \mathbb{N} \\
& j=1, \mathbb{N}
\end{aligned}
$$

Upon inverting Equation (2-A), values of $\varphi$ i can be calculated

$$
[\varphi]=[b]^{-1}[A]
$$

Although this is a simple method to use, a problem arise in selecting the energies $E_{2}, E_{3}, \ldots, E_{N}$. In fact, the main disadvantage of this method is that there are no logical methods for choosing $E_{1}$, ..., $E_{N}$. If it were to be used, the best scheme according to Ringle* is to choose a variety of energy arrays, to calculate the absolute value of ( $\operatorname{det}[b])$ for each array, and then to use the array which yields the largest value of $|\operatorname{det}[b]|$.
c) Polygonal method $\quad \varphi(E)$ is represented by straight lines (23). The method assumes that the actual spectrum can be represented by a set of $N$ values for the flux corresponding to points $E_{1}, E_{2}, \ldots, E_{N}$. $N$ is the number of reactions used. It is further assumed that within each energy interval the flux varies linearly.

As in the Step Function Approximation, the number of flux constants and energy points must equal $N$. The energy point $E_{1}$ is chosen as the Iowest energy at which activation can occur for any reaction, with $E_{N}$ being the energy above which either $\Phi(E)$ or $\Theta^{\prime}(E)$ is zero.

The flux $\varphi(E)$ between $E_{1}$ and $E_{2}$ is

$$
\begin{align*}
& \varphi(E)=\varphi_{1}+\left(\frac{Q_{2}-Q_{1}}{E_{2}-E_{1}}\right)\left(E-E_{1}\right)= \\
& =\varphi_{2}\left(\frac{E-E_{1}}{E_{2}-E_{1}}\right)+\varphi_{1}\left(\frac{E_{2}-E}{E_{2}-E_{1}}\right) \tag{3}
\end{align*}
$$

* See page 28.
and
$\varphi(E)=\varphi_{N}\left(\frac{E-E_{N-1}}{E_{N}-E_{N-1}}\right)+\varphi_{N-1}\left(\frac{E_{N}-E}{E_{N}-E_{N-1}}\right)$
with $E_{N-1} \leqslant E \leqslant E_{N} \quad$ and $\quad i=1, N$

Inserting Equations (3) and (3-A) in Equation (1), the following expression is obtained:
$A_{s p i}=\int_{E_{1}}^{E_{2}}\left[\varphi_{2}\left(\frac{E-E_{1}}{E_{2}-E_{1}}\right)+\varphi_{1}\left(\frac{E_{2}-E}{E_{2}-E_{1}}\right)\right] \widetilde{\sigma i}_{i}(E) d E+$
$+\int_{E_{2}}^{E_{3}}\left[\varphi_{3}\left(\frac{E-E_{2}}{E_{3}-E_{2}}\right)+\varphi_{2}\left(\frac{E_{3}-E}{E_{3}-E_{2}}\right)\right] \sigma_{i}(E) d E+\ldots+$
$+\int_{E_{N-1}}^{E_{N}}\left[\varphi_{N}\left(\frac{E-E_{N=1}}{E_{N}-E_{N-1}}\right)+\varphi_{N-1}\left(\frac{E_{N-}-E}{E_{N}-E_{N-1}}\right)\right] \theta_{i}(E) d E$

The set of Equations ( $3-B$ ) can be written in matrix notation, thus
$[A]=[b]\left[\begin{array}{l}\varphi \\ \text { In this } \\ \text { case the terms of matrix }\end{array}[b]\right.$ are given by

$$
\begin{aligned}
& b_{i j}=\frac{1}{E_{j}-E_{j-1}} \int_{E_{j-1}}^{E_{j}} E \sigma_{i}(E) d E-\frac{E_{j-1}}{E_{j-} E_{j-1}} \int_{E_{j-1}}^{E_{j}} \sigma_{i}(E) d E+ \\
&+\frac{E_{j+1}}{E_{j+1}-E_{j}} \int_{E_{j}}^{E_{j+1}} \sigma i(E) d E-\frac{1}{E_{j+1}-E_{j}} \int_{E_{j}}^{E_{j+1}} E \sigma i(E) d E \\
& E_{i o r}=1, N \\
& j=2, N-1
\end{aligned}
$$

These coefficients can be calculated by selecting the energy intervals $E_{1}, \ldots, E_{N}$ and using the cross section data for each reaction. Once the elements of $[b]$ are determined, $[\varphi]$ can be obtained by solving the matrix equation.
$[\hat{Q}]=[b]^{-1}[A]$

A comparison of experimental results using this method with the results of a 20 group calculation indicated that for neutron energies below 350 KeV , the polygonal method provides good agreement. However, for energies above 350 KeV the results are not satisfactory (22). This method suffers from the same disadvantages as the Step Function Approximation method.
d) Simple nolynomial method In this method $\varphi(E)$
is represented by a polynomial in $E$ (23)
$\varphi(E)=a_{0}+a_{1} E+\ldots+a_{N-1} E^{\mathbb{N}-1}$

Using $N$ reactions, the coefficients $a_{0}, a_{1}, \ldots, a_{N-1}$ can be determined in a polynomial of degree $\mathrm{N}-1$.

Substituting the above expression in Equation (1),
$A_{\text {spi }}$ becomes

$$
A_{s p i}=a_{0} \int_{\operatorname{Bmin}}^{\max } \sigma i(E) d E=a_{1} \int_{\operatorname{Emin}}^{\operatorname{Bmax}} E \sigma i(E) d E+\ldots+
$$

$$
+a_{\mathbb{N}-1} \int_{\mathrm{Emin}}^{\mathrm{Emax}} E^{\mathrm{N}-1} \theta_{i}(E) d E
$$

or, using matrix notation
$[A]=[b][a]$
with the solution

$$
[a]=[b]^{-1}[A]
$$

where
$b_{i j}=\int_{\operatorname{Bnin}}^{\max } E^{j-1} \sigma_{i}(E) d E$

Reports on the use of this method are rare. Other polynomial methods, yet to be discussed, have been employed more frequentiy.
e) Expansion in orthonormal combinations of cross sections The spectral distribution $\varphi(E)$ is represented by a series of orthonormal energy dependent functions $\psi_{i}(E)$ (24, 25). Thus

$$
\begin{equation*}
\varphi(E)=a_{1} \psi_{1}(E)+a_{2} \psi_{2}(E)+\ldots+a_{N} \psi_{N}(E)=\sum_{i=1}^{N} a_{i} \psi_{i}(E) \tag{5}
\end{equation*}
$$

where $\mathbb{N}$ is the number of detector reactions used, and $\psi_{i}(E)$ are linear combinations of the cross sections $\sigma_{i}(E)$
$\psi_{1}(E)=b_{11} \sigma_{1}(E)$
$\psi_{2}(E)=b_{21} \sigma_{1}(E)+b_{22} \sigma_{2}(E)$
;

$$
\begin{equation*}
\psi_{N}(E)=n_{N} \sigma_{1}(E)+\ldots+b_{N N} \sigma_{N}(E) \tag{6-C}
\end{equation*}
$$

or

$$
\begin{equation*}
[\psi]=[b][\sigma] \tag{7}
\end{equation*}
$$

This leads to $\frac{N(N+1)}{2}$ constant coefficients $b_{j k}$ and $\frac{N(N+1)}{2}$ non duplicated orthonormal condition equations with which $b_{j k}$ are to be evaluated.

The coefficients $b_{j k}$ can be calculated from an orthonormality condition

$$
\int_{0}^{\infty} \psi_{j}(E) \psi_{k}(E) d E=\delta_{j k}
$$

In fact, multiply Equation (6-A) by $\psi_{1}(\mathbb{E})$ and integrate

$$
\int_{0}^{\infty} \psi_{1}^{2}(E) d E=\int_{0}^{\infty} b_{11} \sigma_{1}(E) \psi_{1}(E) d E=b_{11} \int_{0}^{\infty} \sigma_{1}^{2}(E) d E=1
$$

Since the cross sections are known $b_{11}$ can be calculated.
Multiplying Equation ( $6-B$ ) by $\psi_{1}(E)$ and $\psi_{2}(\mathbb{E})$ and integrating, the following expressions are obtained:

$$
\begin{aligned}
\int_{0}^{\infty} \psi_{1}(E) \Psi_{2}(E) d E= & b_{11} b_{21} \int_{0}^{\infty} \sigma_{1}^{2}(E) d E+b_{11} b_{21} \int_{0}^{\infty}\left(\sigma_{1}^{2}(E)\right. \\
& \left.+\sigma_{2}^{2}(E)\right) d E=0 \\
\int_{0}^{\infty} \psi_{2}^{2}(E) d E= & b_{21}{ }^{2} \int_{0}^{\infty} \sigma_{1}^{2}(E) d E+2 b_{21} b_{22} \int_{0}^{\infty} \sigma_{1}(E) \sigma_{2}(E) d E+ \\
& +b_{22} 2^{0} \int_{0}^{\infty} \sigma_{2}^{2}(E) d E=1
\end{aligned}
$$

From these two equations $b_{21}$ and $b_{22}$ are determined.
In general, writing the equations in a matrix notation, the following equations are obtained:

$$
\begin{equation*}
[\Phi]=[a][\psi] \quad \text { and }[\psi]=[b][\theta] \tag{8}
\end{equation*}
$$

where

$$
[\varphi]=\varphi(E)
$$

$[a]=\left[a_{1}, a_{2}, \ldots, a_{\mathrm{N}}\right]$
$[\Psi]=\left|\begin{array}{l}\psi_{1}(E) \\ \psi_{2}(E) \\ \vdots \\ \psi_{N}(E)\end{array}\right|$
$[\theta]=\left|\begin{array}{l}\sigma_{1}(E) \\ \sigma_{2}(E) \\ \vdots \\ \sigma_{N}(E)\end{array}\right|$
$[b]=\left|\begin{array}{lllll}b_{11} & 0 & 0 & \cdots & 0 \\ b_{-11} & b_{22} & 0 & \cdots & 0 \\ b_{31} & b_{32} & b_{33} & \cdots & 0 \\ b_{N 1} & b_{N 2} & b_{N 3} & \ldots & b_{N N}\end{array}\right|$
The inverse of Equation (8) is
$[\sigma]=\left[\begin{array}{l}b \\ {\left[-1\left[\begin{array}{l}\psi \\ \end{array}\right]=[c][\psi]\right.} \\ \text { with }[c]=[b]-1\end{array}\right.$.
The nth equation of the expression (8-A) can be written as $\sigma_{n}(E)=\sum_{I=1}^{n} c_{n I} \psi_{I}(E)$

Substituting this and Equation (5) in Equation (1), the spewcific activation becomes:
$A_{s p n}=\int_{0}^{\infty} \sum_{i=1}^{N} a_{i} \psi_{i}(E) \sum_{I=1}^{n} c_{n l} \Psi_{I}(E) d E$
or

$$
\begin{aligned}
A_{s p n} & =\sum_{i=1}^{N} a_{i} \sum_{l=1}^{n} c_{n l} \int_{0}^{\infty} \Psi_{l l}(E) \psi_{I}(E) d E \\
& =\sum_{i=1}^{N} a_{i} \sum_{l=1}^{n} c_{n I} \delta_{i l}=\sum_{l=1}^{n} c_{n I} a_{I}
\end{aligned}
$$

or in matrix notation

$$
[A]=[c][a]
$$

Upon inverting the above equation

$$
\left[\begin{array}{l}
c
\end{array}\right]-1[A]=[a]
$$

$$
\text { and knowing that }[c]^{-1}=[b],[b] \text { becomes }
$$

$$
[b]=[a][A] \quad \operatorname{end}[a]=[b][A]-i
$$

The flux spectrum is then found from

$$
\Phi(E)=\sum_{i=1}^{N} a_{i} \psi_{i}(E)=\sum_{i=1}^{N} a_{i} \sum_{j=1}^{i} a_{i j} E^{i}
$$

Comparing the Orthonormal Combinations of Cross Sections method with other unfolding methods, Ringle (26) states that this method provides accurate neutron flux spectrum determination and concludes that its use should be seriously considered.
g) Successive exponentials method In this method, the neutron energy is divided into suitable intervals. The flux $\phi(E)$ in each interval is assumed to have the form (11) $\varphi(\mathbb{E})=A \exp (-k E)$

The values of $\sigma_{i}(E)$ within each interval are the values reported in the literature except for the energy interval where $\sigma_{i}(E)$ starts to become different from zero. In this case $\sigma_{i}(E)$ is assumed to be a straight line
$\sigma_{i}(E)=a_{i}+b_{i} E$
Bresesti (27) states that the use of linearized cross sections, as represented by Equation (9), is a source of error for this method. This is particularly true for detectors such as $238_{\mathrm{U}}$ Which do not have an energy dependent cross section that can be represented by a straight line.
2. Perturbation methods

These methods assume that $\varphi(\mathbb{E})$ differs only slightly from a. fission spectrum. Thus, the main objective of the Perturbation methods is to determine parameters which characterize the Qeviation of $\phi(\mathbb{E})$ from a fission spectrum. The representation of the actual flux may also include a weighting function (as is the case vith weighting methods) to improve the representation

Of the spectrum. Four examples of Perturbation methods are described in the following paragraphs. These are: a) SemiEmpirical Deviation b) Polynomial Deviation Function c) Orthonormal Deviation Function d) Spectral Indices.
a) Semi-cmuirical deviation method The spectral distribution $\varphi(E)$ is expressed as the product of a fission spectrum and a function $f(B, E)$ which corrects for the deviation of the actual spectrum from a fission spectrum represented by $S(E)$. $B$ is a constent discussed below.
$\varphi(E)=S(E) f(B, E)$
In case of an infinite medium with uniformly distributed neutron sources

$$
\varphi(E)=\frac{S(E)}{\Sigma(E)}
$$

where $S(E)$ is the source density and $\Sigma(E)$ denotes a macroscopic removal cross section. $\Sigma(E)$ is taken to be the sum of a constant term $\Sigma_{c}$, which represents inelastic scattering and absorption of neutrons in all materials except hydrogen, and $\Sigma_{H}(E)$ which takes into account the elastic scattering by hydrogen

$$
\Sigma(E)=\Sigma_{C}+\Sigma_{H}(E)
$$

Witnin the energy range from 2 to $12 \mathrm{MeV}, \sigma_{\mathrm{H}}(E)$ can be represented by the following empirical relation (11)

$$
\sigma_{H}(\Xi)=5.13 \mathrm{E}^{-0.725}
$$

where E is expressed in MeV and $\sigma_{\mathrm{H}}(\mathrm{E})$ in barns. Thus
$\Phi(E)=\frac{S(E)}{\Sigma_{C}+\Sigma_{H}(E)}=\frac{S(E)}{\Sigma_{C}\left(1+B E^{-0.725}\right)}$
where $S(E) / \Sigma_{c}$ is a fission flux distribution divided by a constant, and $1 / 1+B E^{-0.725}$ is the energy dependent perturbation parameter.

The procedure for evaluating B, proposed by Dietrich in (11), is to calculate the response integrals of several threshold detectors for different values of $B$, to determine the corresponding experimental values of these response integrals, and finally to select the value of $B$ which gives the best agreement.

This method is simple, but its application is limited to homogeneous reactor cores or to their immediate surroundings.
b) Polynomial deviation function In this method the deviation of the spectrum from a fission spectrum is repre= sented by a polynomial

$$
\varphi(E)=\varphi_{f i s}(E)\left(a_{0}+a_{1} E+\ldots+a_{N} E^{N}\right)
$$

The number of the threshold detectors used is equal to $\mathbb{N}+1$ so
 unknowns. The calculation procedure is similar to the one described for the simple polynomial method.
c) Orthonormal deviation function The spectral distribution is represented by a fission spectrum times an orthonor-
mai combination of simple polynomials (28)
$\varphi(\Xi)=\varphi_{\text {fis }}(E) \sum_{i=1}^{M} a_{i} \psi_{i}(E)$

For $\varphi_{\text {fis }}(\mathbb{E})=1$ this method collapses to the method of Expansion in Ortionormal Combinations of Simple Polynomials (11).
d) Spectral indices In this method $\varphi(E)$ is represented as
$\varphi(D)=C E^{\frac{1}{2}} \exp (-\theta E)$
Where $E$ is in MeV and $\theta$ in $\mathrm{MeV}^{-1}$. $\theta$ is to be adjusted from the fission disiribution value of 0.775 to one which is more representative of the response integral values. The distribution has its maximum at
$\max =\frac{1}{2 \theta}$
The spectral index, $S_{i j}$, as defined by Grundl and Usner (8) is
$S_{i j}=\frac{A_{i}(E)}{A_{j}(E)}=\frac{\int_{0}^{\infty} \sigma_{i}(E) \varphi(E) d E}{\int_{0}^{\infty} \sigma_{j}(E) \varphi(E) E}=\frac{\bar{\sigma}_{i}}{\overline{\sigma_{j}}}$
mere i and j refer to different reactions. The procedure Ecr the determination of $\theta$ is similar to the one described for উhe Somi-mpirical Deviation method. Both experimental and cinulationsi velues of $S_{i j}$ are determined for several values oき $\theta$. The value of $\theta$ which gives the best agreement between
the experimental and calculational values is the one to be used as the deformation parameter.

## 3. Weighting methods

The spectral distribution is represented by a mathematical expression such as polynomials, exponential, etc. However, because an improved representation of the spectrum can be obtained by employing a weighting function, one is included. The Relative Deviation Minimization Method (RDMM) provide a good example of spectrum representation using weighting methods.

RDMM, as proposed by Di Cola (17), assumes the following expression for the unknown spectrum:

$$
\varphi(E)=W(E) \sum_{i=1}^{N} a_{i} \psi_{i}(E)
$$

where $W(\mathbb{E})$ is a weghting function which can be 1 , the fission spectrum, $e^{-a E}$ or other functions. $\Psi_{i}(\mathbb{T})$ are a series of linearly independent functions.

The best approximation to $\varphi(E)$ is a function, $\vec{\Phi}_{m}(\mathbb{E})$ which minimizes the following quadratic form:

$$
Q\left(m, a_{1}, \ldots, a_{i j}\right)=\sum_{j=1}^{N}\left[\frac{A_{s p j}-\int_{0}^{\infty} \widetilde{\widetilde{j}_{j}}(E) \bar{क}_{m}(E) d E}{A_{S V j}}\right]^{2}
$$

where $\bar{\varphi}_{\mathrm{m}}$ is given by

$$
\bar{\phi}_{m}(\mathbb{E})=W(E) \sum_{i=1}^{m} a_{i} \psi_{i}(W)
$$

with $m \leqslant N$ and $N$ being the number of reactions used.
Minimizing $Q\left(m, a_{1}, a_{2}, \ldots, a_{N}\right)$, for a given $m$, involves differentiation of $Q\left(m, a_{1}, a_{2}, \ldots, a_{N}\right)$ with respect to $a_{K}$ as follows
$\frac{\partial Q}{\partial a_{k}}=0 \quad k=1, \ldots, m$
The result is a set of $m$ linear equations with $m$ unknowns. In matrix form

$$
[S]^{T}[S][a]=[S]^{\mathbb{T}}[1]
$$

where $S_{i j}$ and $S_{i j}^{T}$ are

$$
\begin{array}{ll}
S_{i j}=\frac{1}{A_{s p i}} \int_{0}^{\infty} \sigma_{i}(E) W(E) \psi_{j} d E & \begin{array}{l}
i=1, \ldots, N \\
j=1, \ldots, m
\end{array} \\
S_{i, j}^{T}=\frac{1}{A_{s p j}} \int_{0}^{\infty} \sigma_{j}(E) W(E) \Psi_{i} d E & \begin{array}{l}
i=1, \ldots, m \\
j
\end{array} \quad 1, \ldots, N
\end{array}
$$

For each value of $m \leqslant N$, there is a solution. Among these solutions, the one that minimizes $Q$ is the best approximation to the spectral distribution $\varphi(\mathbb{E})$ 。

In order to establish the dependence of RDMM on the perticular choice of $\psi_{i}(E)$, Di Coia ard Rota in (27) tested this method with experimental and theoretical datia. They used exp $(-\mathbb{F})$ as the weighting function end selected the following types of polynomials for $\psi_{i}(E)$ :

- Simple polynomials
- Orthonormal polynomials
- Laguerre polynomials
- Chebyshev polynomials

Theoretical data consisted of computer calculations of theoretically constructed "test data", while experimental data involved the following eight detectors: ${ }^{237} \mathrm{~Np},{ }^{238} \mathrm{U}_{\mathrm{U}},{ }^{232_{\mathrm{Th}}}$, ${ }^{32} \mathrm{~S},{ }^{58} \mathrm{Ni},{ }^{56} \mathrm{Fe},{ }^{27} \mathrm{Al}(\mathrm{n}, \mathrm{p})$, and ${ }^{27} \mathrm{Al}(\mathrm{n}, \alpha)$.

Significant differences were observed for high energy values of the experimental data. However, these differences did not appear when "test data" were used. This indicated thet the results of RDMM can be considered as independent of the type of $\Psi_{i}(E)$ functions used. Furthermore, RDMM data with experimental results indicated that for $m=5,6,7$, the values of $Q$ were not appreciably different from each other. This would indicate that a greater number of $\Psi_{i}(E)$ functions would not improve the final results.
4. Comparison of methods

Within the energy range of about 2 to 30 MeV , Ringle (26) compared the following methods:

- Flux Integral method
- Step Curve method
- Polyzonal method
- Excansion in Orthonormal Combination of Cross Sections
- Legencire Polynomial Expansion metnod
- Fourier Expansion method

The last three methods are examples of the Orthoconal Polynomial Expansion. In order to compare the above methods, ten different "trial spectra" were selected. These spectra included:
$\varphi_{1}(E)=0.7$
$\varphi_{2}(E)=1-0.05714 E$
$\varphi_{3}(E)=\exp (-0.115 E)$
and other spectra such as step curves and oscillating curves. For each method, the cross sections of the following reactions were used:
$5^{58}(n, p){ }^{58} \mathrm{Co}$
${ }^{31_{P}}(n, 2){ }^{31} \mathrm{Si}$
${ }^{56} 6_{\mathrm{Fe}}(\mathrm{n}, \mathrm{p}){ }^{56_{\mathrm{Mn}}}$
${ }^{24} \mathrm{Mg}(\mathrm{n}, \mathrm{p}){ }^{24 \mathrm{Na}}$
${ }^{203}{ }_{T 1}(n, 2 n){ }^{202}{ }_{T 1}$
${ }^{63} \mathrm{Cu}(\mathrm{n}, 2 \mathrm{n}){ }^{62_{\mathrm{Cu}}}$
Using the above spectra and cross sections, the activities were calcuiated and used as inputs for each method under investigation. The calculated spectra were then compared with the trial spectra. Based on these comparisors, Ringle made the following observations:
a) Flux Integral method, being sensitive to the choice of Eeff is too subjective and should not be considered for anplication.
b) Siep Curve and Polyronal methods provide results which are dependent upon the choice of energy intervals. They are the only methods which represent the spectra by straight line segments. The best polygonal results were clearly better than the best results obtained by means of the Step Function method.
c) Cross Sections Expansion provided results which agreed well with experiment for all trial spectra.
d) The Legendre Expansion method provided good agreement with all trial spectra except for the step function fluxes.
e) The Fourier Expansion method provided generally poor agreement with the trial spectra.

In view of these results, Ringle (25) concluded that "Three methods give good results: the Cross Section Expansion method, which is good for both step function and continuous fluxes; the Legendre method, which is good for continuous fluxes and poor for step function fluxes; and the Polygonal method, which is reasonably good for both types of fluxes. OI these three, the Cross Section Expansion method is the best."

Other intercomparisons were accomplished by severel investisators. Bresesti (27) noted that if the actuel shape of $\varphi(\mathbb{E})$ is significantly different from the proposed trial spectrum, Step Function Approximation and Polygonal methods provide poor results. The method of Successive Exponentials provided unacceptable results in the first energy interval. This was

Gue to the fact that a linearized cross section shape does not adequately represent the energy variation of the cross section of the ${ }^{238}{ }_{U}$ detector. The method of Spectral Indices, with one parameter, provided satisfactory results only at energies greater than 2 MeV (28).
III. COMPUTER CODES FOR SPECTRUM UXIFOLDING

Within recent years, several computer codes have been developed to unfold neutron spectra from the measured activities of a set of activation foils ( $29,30,31$ ). The measured specific activity $A_{s p i}$ (or response integral) of activation reaction i is related to the spectrum by Equation (10)
$A_{s p i}=\int_{0}^{\infty} \sigma_{i}(\mathbb{E}) \varphi(E) d E \quad i=1,2, \ldots N$
Generally, the unfolding technique incorporated into the computer code is produced by the following procedures:

1. The energy domain of $\varphi(E)$ is divided into $M$ energy intenvals; M being genereliy much greater than $N$, the number of detector reactions to be activated.
2. Equations of type (10) are approximated by a series of linear equations

with each energy interval represented by a particular value of j.
3. A trial spectrum, based upon some a priori knowledse of $\varphi(\Xi)$, is used as the initiel or trial value for $\varphi(\mathbb{B})$. Detector foil activations are calculated, based on the trial spectrum. These
calculations are compared with the experimentally determined activities produced by the actual neutron flux. 4. The computer codes modify the trial spectrum so that, within experimental errors, measured activities equal calculated activities.

As mentioned earlier, the number of energy intervals $M$ is generally greater then $\mathbb{N}$, the number of reaction used. This means that the solution spectrum is not unique, and that portions of it will depend entirely on such input data as the trial spectrum, cross sections, etc.

Several spectrum unfolding codes are available; the more widely used codes are SPECTRA (developed in 1966), SAMD-II (1967), and CRYSTALL BALL (1974). A brief description of SPECPRA and CRYSTAIL BALI as well as a more detailed description of the SAND-II code are given below.

## A. SFECTRA

This method, developed by Greer and Walker (32), Greer et al. (33), is based upon a Perturbation method in which an initial trial spectrum is assumed end then modified with successive iterations so that it will agree with the activation data. The flux evaluated with the SFPCTRA code represents the flux which minimizes the least squares error between the measured and calculated activities.

Let $\varphi_{1}, \varphi_{2}, \ldots, \varphi_{\mathrm{N}}$ correspond to erergy values $\mathrm{E}_{1}, \Xi_{2}$, $\ldots, \mathbb{M}_{\mathrm{M}}$, respectively. Then $\underline{q}(\mathbb{B})$, ir the interval $\mathrm{E}_{\mathrm{k}}$ to $\mathrm{E}_{\mathrm{k}+1}$,
is given by
$\phi(\Xi)=\varphi_{k+1}\left(\frac{E-E_{k}}{E_{k+1}-E_{k}}\right)+\varphi_{k}\left(\frac{E_{k+1}-E}{E_{k+1}-E_{k}}\right)$

It is assumed that there is no flux below $E_{q}$ and that the flux at $\mathrm{E}_{\mathrm{M}}$ is zero, i.e, $\varphi_{\mathrm{M}}=0$. Substitution of Equation (11) into Equation (10) yields

$$
\begin{align*}
A_{s p i} & =\int_{E_{1}}^{\mathbb{E}_{1}}\left[\varphi_{2}\left(\frac{E-E_{1}}{E_{2}-E_{1}}\right)+\varphi_{1}\left(\frac{E_{2}-E}{E_{2}-E_{1}}\right)\right] \sigma_{i}(\mathbb{E}) d E+ \\
& +\int_{E_{2}}^{E_{3}}\left[\varphi_{3}\left(\frac{E-E_{2}}{E_{3}-E_{2}}\right)+\varphi_{2}\left(\frac{E_{3}-E}{E_{3}-E_{2}}\right)\right] \sigma_{i}(E) d E+\ldots+ \\
& +\int_{E_{M-1}}^{E_{M}}\left[\varphi_{M}\left(\frac{E-E_{M-1}}{E_{M}-E_{M-1}}\right)+\varphi_{\mathbb{M}-1}\left(\frac{E_{M}-E}{E_{M}-E_{M-1}}\right)\right] \sigma_{i}(\mathbb{E}) d E \tag{12}
\end{align*}
$$

In Iquation (12) i $=1,2, \ldots, N$, where $N$ is the number of activation reactions used and $N \leqslant M$. This set of equations cen be writien as a matrix equation
$[\mathrm{i}]=[\mathrm{c}][\mathrm{q}]$
where

$$
[A]=\left|\begin{array}{c}
A_{1} \\
A_{2} \\
\vdots \\
A_{M}
\end{array}\right| \quad[\varphi]=\left|\begin{array}{l}
\varphi_{1} \\
\varphi_{2} \\
\vdots \\
\varphi_{N}
\end{array}\right|
$$

and [C] is an $M \times N$ matrix whose entries are defined by the following expression:

$$
c_{j i}=\int_{E_{i-1}}^{E_{i}} \frac{E-E_{i-1}}{E_{i}-E_{i-1}} \sigma_{j}(E) d E+\int_{E_{i}}^{E_{i+1}} \frac{E_{i+1}-E}{E_{i+1}-E_{i}} \sigma_{j}(E) d E
$$

For $C_{j 1}$ the velue $E_{0}$ is defined to be $E_{1}$ and for $C_{j N}, E_{N+1}$ is defined to be $\mathrm{E}_{\mathrm{N}}$.

In order to solve Equation (11), it is assumed that the rank of $[C]$ is $M$ and the first $M \times N$ submatrix of [C] is nonsingular. Denoting the calculated activities by $A_{C}$, then the least squares error E between the measured activity A and the calculated activity $A_{C}$ is
$[E]=\left(\left[A_{C}\right]-[A]\right)^{T}\left(\left[A_{C}\right]-[A]\right)$
B is minimum with respect to $\varphi$ when
$\frac{\partial E}{\partial \phi}=0$
$\frac{\partial^{2} E}{\partial \varphi^{2}}>0$
Thus,
$\frac{\partial E}{\partial \varphi}=2[C]^{T}\left(\left[A_{C}\right]-[A]\right)=2\left([C]^{T}[C][\varphi]-[C]^{T}[A]\right)=0$
If $[c]^{T}[c]$ is nonsingular,
$[\varphi]=\left([C]^{T}[C]\right) / 2[C]^{T}[A]$
this is a valid solution only when the number of energy intervals is equal to the number of reactions used. However, when $N$ is greater then $M$, the matrix $[C]^{T}[C]$ is singular so no solution can be obtained directly. In this case a best guess spectrum, denoted by $\left[\varphi_{0}\right]$, is selected to represent the initial values of $[0]$ and a new error function is defined as follows:
$\left[\Xi_{1}\right]=\left(\left[A_{C}\right]-[A]\right)^{T}\left(\left[A_{C}\right]-[A]\right)=\left([\varphi]-\left[\varphi_{0}\right]\right)^{T}\left([\varphi]-\left[\varphi_{0}\right]\right)$

The new error function $\left[\mathbb{E}_{1}\right]$ is the sum of the least squares erwor between $[A]$ and $\left[A_{C}\right]$ pius the least squares error between a solution and the initiol guess solution. The new function $\left[\mathbb{F}_{1}\right]$ is minimal with respect to $\varphi$ when the first and second pertiel derivatives of $\left[E_{1}\right]$ respect to $\varphi$ are 0 and $>0$, respectively.
$\frac{\partial \Xi_{1}}{\partial \varphi}=2 C^{T}\left(\left[A_{C}\right]-[A]\right)+2[I]^{T}\left([\varphi]-\left[\varphi_{0}\right]\right)=0$
$\frac{\partial^{2} \mathbb{S}_{1}}{\partial \varphi^{2}}=[C]^{T}[C]+[I]$
If $[C]^{T}[C] \div[I]$ is positive definite, then a unique solution to تouation (13) exists and $\varphi$ can be determined.

The above technique is incorporated in the SFECTRA code, which is capable of computing differential flux up to a maximum of 50 energy points. The code contains a cross section library of 28 reactions, taken from the compilation of McElroy (14).

## B. CRYSTALI BALI

The code, developed by Kam and S.tallman (34), is based on a direct approximation of integral equations by linear combination of integral operators. In this method the measured specific saturation activities $A_{\text {spi }}$ are related to the unknown spectrum $\varphi(\mathbb{B})$ through the integrai

$$
\begin{equation*}
A_{s, i}=\int_{0}^{\infty} \phi(\mathbb{E}) \sigma_{i}(E) d E+\ddot{E}_{i} \quad i=1,2, \ldots, N, \tag{14}
\end{equation*}
$$

where $E_{i}$ are measuring errors which have an estimated variance $\nu$ ㄹ derineà as

The method assumes that the general shape of $\varphi(E)$ is close to an a priori estimate of the actual spectrum. Denoting with $\Psi(E)$ this trial spectrum, the solution spectrum has to minimize $s^{2}$
$S^{2}=\int_{0}^{\infty}\left[\frac{d}{d E} \frac{\varphi(E)}{\psi(E)}\right]^{2} W(E) d E$
where $W(E)=\frac{k}{E}$ and $k$ is a constant. Furthermore, the solution is subject to the condition that the weighted mean square error remains below a given tolerance limit $f$
$\sum^{\mathbb{N}}\left(\frac{\ddot{E}_{i}}{\nu_{i}}\right)^{2}<f$
$i=1$

The trial spectrum in Equation (15) can be written as $\psi(E)=\varphi(E)+F(E)$
where $F(E)$, the spectrum deviation term, is the difference between the actual and the trial spectrum. The method assumes that the trial spectrum, being a close approximation to the true spectrum, can be expressed as a linear combination of the measured activities $A_{s p i}$ as follows:
$\psi(\mathbb{E})=\varphi(\mathbb{S})+F(E)=\sum^{N} C_{i} A_{\text {spi }}$

Therefore,
$F(\mathbb{E})=\sum_{i=1}^{N} C_{i}\left[\int_{0}^{\infty} \varphi(E) \sigma_{i}(E) E+\dot{H}_{i}\right]-\varphi(E)$
or using Dirac's $\delta$-function $F(\mathbb{I})$ can be written as
$F(\mathbb{E})=\int_{0}^{\infty}\left[\sum_{i=1}^{\mathbb{N}} c_{i} \sigma_{i}(E)-\delta\left(E-E^{\prime}\right)\right] \varphi\left(E^{\prime}\right) d E^{\prime}+\sum_{i=1}^{N} C_{i} E_{i}$
The problem of solving the activation equation with respect to $\Phi(\mathbb{E})$ becomes the problem of minimizing $F(\mathbb{E})$.

Skipping the mathematical elaborations (35), the true spectrum $\varphi(E)$ at a given energy $E_{j}$ is calculated to be

$$
\tilde{\varphi}\left(\Xi_{j}\right)=\psi\left(\Xi_{j}\right) \sum_{i=1}^{N} c_{i j} R_{i}
$$

where $R_{i}$ is the ratio of measured to calculated activity. Kam and Stallmen arrive at an expression for the difference between $\varphi(\Xi)$ and $\tilde{\varphi}(\Xi)$ which is a linear functional in terms of $\frac{d}{d \Xi} \frac{\rho(\mathbb{E})}{\psi(\mathbb{B})}$
 is equivalent to minimizing $D$, defined as

$$
D=s^{2}+g^{2} f
$$

where $g$ determines how far the tollerance limit f is to be reduced within the approximation.

The algorithm consists of an iterative procedure in which an output spectrum for a given $g$ is used as input spectrum in the next step. Numerical experience obtained by the authors has shown that the output spectrum is relatively insensitive to the choice of the initial estimate of the spectrum. However, this does not apply to the energy intervals which are poorly covered by the detector response. In energy regions not covered by the detectors, the unfolded spectrum is solely determined by the input spectrum. This does not necessarily mean that the solution will be equal to the trial spectrum: The solution spectrum in these energy regions may be a multiple of the trial spectrum, or changed in such a way as to provide a smooth transition from the input spectrum to the solution spectrum. The latter applies to such other codes as SPECTRA 2 Ủ SAMD-II as well.

> C. SAITD-II

This code developed by Mcilroy and collaborators (14), provides a best fit spectrum for a given input set of "infinitely dilute" foil activities. The term "infinitely cilute"
means that the density of target nuclei is small enough to preclude such effects as self-absorption and self-shielding. The measured input activities are corrected to specific activities and expressed in disintegrations per second per target nucleus. The energy range of the solution spectrum is from $10^{-10} \mathrm{MeV}$ to 18 MeV , divided in 620. There are 45 intervals per decade up to 1 MeV , and 170 intervals between 1 and $18 \mathrm{MeV}^{*}$. The problem is essentially to solve for 621 unknown differential flux values in a system of $N$ linear activity equations; $\mathbb{N}$ being the number of reactions used.

The calculational procedure consists of selecting a "best guess" for the initial input spectrum and then iterating to find a final spectrum consistert with the input data. Since the number of reactions used is much smaller than the number of energy intervals, the solution is not unique. Therefore, the appropriateness of the solution depends upon a suitable choice of the initial spectrum (e.g. fission or Maxwellian). The SAND-II iterative procediure involves the following steps: 1. Activities are calculated for each reaction employed. The calculation is based on the current iterative spectrum and an evaiuated cross section Iibrary, which is part of the SAHD-II

[^2]code.
2. The calculated and measured activities for each foil are compared and a correction factor associated with the fiux in the appropiate energy range for each foil is found. This factor, $R_{i}^{(k)}$ is used for the determination of on "activity-veisinted correction term" for the flux in each iteration. 3. A weighting function, $w_{i j}^{(k)}$, is obtained for each foil. This energy dependent function is proportional to the sensitivity function (the product of differential flux and differential cross section) of the foil calculated using the flux determined in the current iteration.
4. Since a number of different foils will have non-zero cross sections in a given energy interval, the weighting functions are combined to obtain on average correction factor at each enerey. The procedure is based on the comparison of measured to colculated activity for each foil and on the relative contribution of the flux, at the given energy, to the activity of a given foil.
5. The average correction factors are then applied to the current iterative flux value at each energy to obtain the next iteraitive flux spectrum.
6. The criterion incorporaied in SAND-II for recognizing an acceptable solution is based on a comparison of successive differential flux iterations. A solution is considered to hare been achieved when the percent aifference between two successive
values is emeller than a specified number, entered as an input to the program.

For a mathematical representation of the procedures, the followiñ symbois are used.
$A_{\text {spi }}=$ measured activit-j for ith foil reaction (specific saturation activity corrected for self-shielding, self-absorption, etc.);
$A_{i}^{(k)}=c a l c u l a t e d ~ a c t i v i t y$ for the ith reaction, based on the kth iterative spectrum;
(k)
$\varphi(E)=$ hth iterative differential flux;
$\mathrm{E}_{j} \quad=$ energy of the ith energy point;
$\Phi_{j}^{(k)}=$ integral inux in the $j t h$ energy interval, between $E_{j}$ and $E_{j+1}$ for the kth iteration;
$\mathcal{G}_{i}(\mathbb{B})=i t h$ foil reaction cross section for a specific neutron interaction;
$A_{i, j}^{(k)}=$ the portion of $A_{i}^{(k)}$ contributed by neutrons in the $j$ th energy interval between $\mathrm{F}_{\mathrm{j}}$ and $\mathrm{E}_{\mathrm{j}+1}$;
$\sigma_{C d}(E)=$ removal cross section for cadmium covering the foils;
${ }^{5} \mathrm{Cd}$ = nuclei density for cacmium covers;
${ }^{Y_{C d}}$ (hichness of cadmium covers;
$j \quad=1,2, \ldots, M$, where H is enery intervel index ( $\mathrm{H}=620$ for original SAMD-II and $M=100$ for the code used in

## this work;

i

$$
\begin{aligned}
i & =1,2, \ldots, N \text { foil index; } \\
k & =1,2, \ldots, \text { iteration index. }
\end{aligned}
$$

Starting from a set of activation equations defined by the Equation (10), the portion of activity contributed by neutrons with energy between $\mathbb{E}_{j}$ and $\mathbb{E}_{j+1}$ (and calculated for fth cadmium covered foil) is

$$
\begin{equation*}
A_{i j}^{(k)}=\int_{E j}^{E_{j+1}} \sigma_{i}(E) \varphi^{(k)}(E) \exp \left(-\mathbb{N}_{C d} X_{C d} \bar{\sigma}_{C d, j}\right)^{d E} \tag{16}
\end{equation*}
$$

In Equation (16) $\bar{\sigma}_{C d, j}$ is cadmium removal cross section averaged over the energy interval $E_{j+1}-E_{j}$ :

$$
\bar{\sigma}_{C d, j}=\frac{\int_{E_{j}}^{E_{j+1}} \sigma_{C d}(E) \varphi^{(k)}(E) d E}{\int_{E_{j+1}}^{E_{j}} \varphi(E) d E} \cong
$$



The approximation in Equation (17) assumes that the $\bar{\sigma}_{C d, j}$ is independent of the kth iterative flux. Errors introduced by the above assumption are negligible when compared to the errors introduced by neglecting the complicated dependence of the neutron scattering effect on the geometry of each specific case. These errors are in turn negligible because of the relative magnitudes of the cadmium attenuation effects.

The values of $\sigma_{i}(E)$ in Equation (16) can be substituted by the interval averaged cross section $\bar{\sigma}_{i j}^{(k)}$
ij

$$
\begin{equation*}
\int_{E_{j}}^{E_{j+1}} \varphi(\mathbb{E}) d E \tag{18}
\end{equation*}
$$

In view of the fine structure of $\sigma_{i}(E)$ for many activation reactions used, the numerical calculation of Equation (18) for
these reactions would require a much finer subdivision of energy interval. For the purpose of calculation, however, it is assumed that the kth iterative differential flux does not vary with energy over the $\mathrm{E}_{\mathrm{j}+1}-\mathrm{E}_{\mathrm{j}}$ interval, that is
$\varphi(E)=\varphi_{j}^{(k)} \quad$ for $\quad E_{j} \leqslant E \leqslant E_{j+1}$
thus Equation (18) reduces to

$$
\bar{\sigma}_{i j}^{(k)} \cong \bar{\sigma}_{i j}=\frac{\int_{j}^{E_{j+1}} \sigma_{i}(E) d E}{\int_{E_{j}}^{E_{j+1}} d E}
$$

The calculation of $\sigma_{i}(E)$, with the assumption expressed in Equation (19), becomes independent of $k$ and Equation (16) becomes
$A_{i j}^{(k)}=\bar{\sigma}_{i j} \int_{E_{j}}^{E_{j+1}} \Phi(E) \exp \left(-\bar{N}_{C d} X_{C d} \bar{\sigma}_{C d, j}\right) d E$
or

$$
\begin{equation*}
A_{i j}^{(k)}=\bar{\sigma}_{i j} \Phi_{j}^{(k)} \exp \left(-\mathbb{N}_{C d} X_{C d} \bar{\sigma}_{C d, j}\right) \tag{20}
\end{equation*}
$$

where $\Phi_{j}^{(k)}$, the integral flux in the $j$ th energy interval is defined as
$\Phi_{j}^{(k)}=\int_{E_{j}}^{E_{j+1}} \varphi(E) d E$
Equation (20) is the calculated activity of the $j$ th energy interval which summed over the 10 energy interval, yields the iterative calculated activity of the itch reaction over the entire energy range
$A_{i}^{(k)}=\sum_{j=1}^{M} A_{i j}^{(k)}$
For instance, at iteration $0 \quad \varphi_{(E)}^{(0)}$ is unperturbed and is equal to the input spectrum) the calculated activity $A_{i}^{(0)}$ is given by
$A_{i}^{(0)}=\sum_{j=1}^{M} \bar{\sigma}_{i j} \Phi_{j}^{(0)} \exp \left(-\mathbb{N}_{1} X_{1} \bar{\sigma}_{1, j}\right)$
$A_{i}^{(0)}$ should be iterated $k$ times so that $A_{i}^{(k)}$ becomes equal to the measured activity $A_{\text {spin }}$, within the specified experimental error.

The mathematical procedure for the flax iteration is based on two veremeters: the activity weighting function $W_{i j}^{(k)}$ and the ratio of measured to calculated activity $R_{i}^{(k)}$, which are defined as follows:
$W_{i 1}^{(k)}=A_{i 1}^{(k)} / A_{s p i}^{(k)}$
(for $j=1$ )
$W_{i K+1}^{(k)}=A_{i M}^{(k)} / A_{\text {SDi }}^{(k)} \quad($ for $j=M)$
$W_{i j}^{(k)}=\frac{1}{2}\left(A_{i j}^{(k)}+A_{i j-1}^{(k)}\right) / A_{s p i}^{(k)} \quad($ for $j=2, \ldots, M)$
and
$s_{i}^{(k)}=A_{\text {spi }} / \hat{A}_{i}^{(L)}$
These parameters, defined by Equations (21) and (22), are combine to give $C_{j}^{(k)}$, the "activity-weighted correction term."
$C_{j}^{(k)}=\sum_{i=1}^{N} W_{i j}^{(k)} \ln R_{i}^{(k)} / \sum_{i=1}^{N} W_{i j}^{(k)}$

Iteration are successively performed according to Equation (23)
$\varphi_{j}^{(k+1)}=\varphi_{j}^{(k)} \exp \left[c_{i}^{(k)}\right]$

If for a given $j: \sum_{i=1}^{N(k)} W_{i j}^{(k)}=0$, log-log interpolation is used to obtain the next ( $k+1$ ) value of differential flux. The interpolation is performed between the nearest lower $j$ and the nearest higher j for which
$\sum_{i=1}^{N(k)} W_{i j} \neq 0$
However, if such $j$ cannot be found, an extrapolation is performed based on one of several alternate forms such as $1 / E$, fission, etc., which is selected as an input option.

If the solution is obtained on the qth iteration, then the relation between $\dot{\phi}_{j}^{(q)}$ (final solution) and $\varphi_{j}^{(0)}$ (initial input) is given by

$$
\begin{equation*}
\varphi_{j}^{(q)}=\varphi_{j}^{(0)} \exp \left[\sum_{p=0}^{q-1} C_{j}^{(p)}\right] \tag{24}
\end{equation*}
$$

Differential flux solutions given by Equation (24) are then integrated to provide the integral flux solution defined by

$$
\Phi\left(\Xi_{j}, E_{M+1}\right)=\sum_{S=j}^{M} \Phi_{S}^{(I)}=\int_{E_{j}}^{E_{M+1}} \varphi(\underline{M}) d E
$$

Given the differential solutions $\varphi_{j}^{(q)}$, the energy limits of sensitivity $E_{i}$ and $E_{i}^{\prime}$ can be evaluated. These limits are defined such that $95 \%$ of the activation of the ith foil is produced by $E>E_{i}$, and the same portion is produced by neutrons of $E<E_{i}^{\prime}$ 。 Equations (25) indicate the mathematical definition of $\mathrm{E}_{i}$ and $\mathrm{E}_{i}^{\prime}$

$=\int_{E_{i}}^{E_{i}^{\prime}} \hat{A}(\Xi) d E=0.95 \int_{\Xi_{i}}^{E_{M+1}} \hat{A}(E) d E$
D. Intercomparison of the Unfolding Codes

In a aathematical intercomperison of SAFD-II, SPECARA, RDM, and two other codes (FARAMSTER and MSSCO), Dierche (36) reported the following general conclusions:
a) The codes recommended for use were either SAND-II or SPECTRA.
b) Equivalent results are obtained from both codes.
c) SAND-II is slightly superior because it is less sensitive to the choice of input spectrum and its computing speed is faster.

In another intercomparison by Dierckx, the performance characteristics of the above codes and CRYSTALI BALL were determined by using each code to analyze the same experimentally obtained activation data (37). The general results of the intercomparison are given in Table 1. In addition, the foilowing conclusions were reported:
a) CRYSTALL BALL and SAND-II are able to calculate the desired spectra fairly well, even with unreasonable input spectra. In such cases, starting with $1 \%$ accuracy in the activation data, solution spectra with errors of $+20 \%$ are obtained. b) Only SPECTRA, SAND-II, and CRYSTALL BALL are able to give moderate spectral details.
c) The unfolding codes recommended for use are SAND-II, SPECTRA, and CRYSTALL BALL.

In order to intercompare the unfolding codes, the IAEA (International Atomic Energy Agency) provided various laboratories with experimental activation data (38). These data were obtained using different fast neutron spectra such as those from Godiva core and a fast breeder mock-up.

Toble 1. Intercomparison of the spectrum unfolding codes

|  | PARAMETERS | RDIMM | MESCO | SPECTRA | CRYSTALL BALL | SAND-II |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| energy range | thermal to fast ( 14 MeV ) |  |  |  |  |  |
| solution model | simple | matrix equations |  |  |  | simple no matrix equations |
| solution | physical bound to mathematical expressions | not always physically acceptable | sometimes no solution is found | lowest $Q^{*}$ avoiding oscillations and negative fluxes |  |  |
| Q attainable | moderate bound to solution imposed |  |  | as low as one likes to have it |  |  |
| sensitivity to trial spectrum | not | great | greatest | small | smallest |  |
| spectral <br> details | few, bound to solution imposed |  |  | moderate bound to broad resolution of detectors |  |  |
| error in the differential spectrum | bound to solution imposed |  |  | $\pm 5 \%$ in each point |  |  |

*Q is the least squares difference between the measured and calculated activities
** in the enercy range covered by the detectors and for detector activity errors of about $\pm 2 \%$ (intercalibrated in a standard spectra)

Since the activation data were not accompanied by either cross section information or input spectra, the intercomparison of the solutions provided by the various codes was very difficult. This is because the general shape of the solution spectrum depends in great measure on the input spectrum. The intercomparison of the results were in part a comparison of input spectra and cross sections and in part a comparison of the characteristics of the codes. In fact, the solution spectra, obtained using different cross section libraries and different "guess spectras" varied greatly。 However, it was concluded that the SAND-II and SPECTRA codes were suitable for spectrum unfolding while CRYSTALI BALI needed further investigation.

Using published experimental activity data, Zijp (39) intercompared four neutron spectrum unfolding codes: CRYSTALL BALL, RFSP-JUL (a slightly modified version of SPECTRA), SAND II, and SANDPET (mainly a SAND-II code incorporating also a Monte Carlo error analysis code). The codes were intercompared using identical activity data, input spectra, and cross sections values. The latter consisted of the ENDF/B-IV dosimetry file and an updated SAND-II library. The results can be summarized as follows:

1. Different unfolding codes do not give the same solution for the same input data. The solutions found were significantly different from each other.
2. There were no energy regions for which all the uniolding codes gave insignificant differences.
3. CRYSTALI BALL introduced the largest modificetion to the input spectrum, and SAND-II the smellest. The modifications seemed to be a smooth function of energy in the case of CRYSTALL BALL, while SAND-II sometimes introduced shary oscillations reminiscent of the resonance region for certain cross sections. It should be noted that, unlike the other codes, SAMD-II runs were performed without using a curve smoothing procedure.
4. Variations of the input activities in accordance with their reported experimental errors give rather small variations in the output spectrum. However, large variations in the solution are observed if cross sections variations are also considered.

## IV. NEUTRON ACTIVATION EQUATIONS

## A. General Equations

The neutron activation equation, in its general form, involves the flux as a function of energy and time: $\varphi=\varphi(\Xi, t)$. To derive the equations, the symbols used in section III, including those redefined below for convenience, will be used: $n \quad=$ number of activation product atom in a given target material at time $t ;$
$m \quad=$ number of target atoms at time $t$ :
$m_{0}=$ number of target atoms at time $t_{0}=0$
$\bar{\theta}_{m n}(t)=$ spectrum-averaged cross section for reaction transmuting the "m" atoms into the "n" atoms;
$\bar{\sigma}_{n}(t)=$ total spectrum-averaged cress section for reactions which remove any of the " n " atoms;
$\Phi(t)=$ time dependent flux $=\int \phi(E, t) d E$
$\lambda_{n}=$ decay constant of the "汭" atoms;
The net time-rate of production of the "n" atoms is

$$
\begin{equation*}
\frac{d n}{d t}=m \bar{\sigma}_{r n}(t) \Phi(t)-n\left(\bar{\sigma}_{n}(t) \Phi(t)+\lambda_{n}\right) \tag{26}
\end{equation*}
$$

where $\bar{\sigma}_{n n}(t)$ and $\bar{\sigma}_{n}(t)$ are defined as follows:

$$
\begin{equation*}
\bar{\sigma}_{x}(亡)=\frac{\int_{0}^{\infty} \sigma_{x}(E) \phi(E, t) d E}{\int_{0}^{\infty} \varphi(\Xi, t) d E} \quad x=n, m n, \ldots, \tag{27}
\end{equation*}
$$

The "burn-up" rate of target atoms is
$\frac{d m}{d t}=-m \bar{\sigma}_{m}(t) \Phi(t)$
with solution
$m=m_{0} \exp -\left[\int_{0}^{t} \bar{\sigma}_{m}\left(t^{\prime}\right) \Phi\left(t^{\prime}\right) d t^{\prime}\right]$

Substituting Equation (28) in Equation (27) yields

$$
\begin{aligned}
\frac{d n}{d t}= & \bar{\sigma}_{m n}(t) \Phi(t) m_{0} \exp \left[-\int_{0}^{t} \bar{\sigma}_{m}\left(t^{\prime}\right) \Phi\left(t^{\prime}\right) d t^{\prime}\right] \\
& -n\left(\bar{\sigma}_{n}(t) \Phi(t)+\lambda_{n}\right)
\end{aligned}
$$

which has the following solution:

$$
\begin{align*}
n(t)= & m_{0} \exp \left[-\lambda_{n} t-\int_{0}^{t} \bar{\sigma}_{n}\left(t^{\prime}\right) \Phi\left(t^{\prime}\right) d t^{\prime}\right] \int_{0}^{t} \bar{\sigma}_{m n}\left(t^{\prime}\right) \Phi\left(t^{\prime}\right) \\
& \exp \left[\lambda t^{\prime}+\int_{0}^{t} \bar{\sigma}_{n}\left(t^{\prime \prime}\right) \vec{\Phi}\left(t^{\prime \prime}\right)-\vec{\sigma}_{m}\left(t^{\prime \prime}\right) d t^{\prime \prime}\right] d t^{\prime} \tag{29}
\end{align*}
$$

If it is assumed that the burn-up of the "m" and the "n" atoms is negligible*, that is

[^3]$$
\bar{\sigma}_{\mathrm{m}}(t) \Phi(t) \ll \lambda_{\mathrm{n}} \quad \text { and } \bar{\sigma}_{\mathrm{n}}(t) \Phi(t) \ll \lambda_{\mathrm{n}}
$$
ther, Equation (29) reduces to
$n(t)=m_{0} \exp \left(-\lambda_{n} t\right) \int_{0}^{t} \bar{\sigma}_{m n}\left(t^{\prime}\right) \Phi\left(t^{\prime}\right) \exp \left(\lambda_{n} t^{\prime}\right) d t^{\prime}$
In the steady-state case, where the flux and cross sections are constant with time, Equation (30) becomes
$n(t)=m_{0} \exp \left(-\lambda_{n} t\right)\left(\bar{\sigma}_{m n} \Phi\right) \int_{0}^{t} \exp \left(\lambda_{n} t^{\prime}\right) d t^{\prime}$
which has the following solution:
$n(t)=\frac{m_{0}}{\lambda_{n}} \bar{\sigma}_{\operatorname{mn}} \Phi\left[1-\exp \left(-\lambda_{n} t\right)\right]$
The activity of the "n" atoms is
$a(t)=n(t) \lambda_{n}$
and the corresponding saturated activity is:
$A^{\prime}(t)=\frac{2(t)}{\left[1-\exp \left(-\lambda_{n} t\right)\right]}=m_{0} \bar{\sigma}_{\mathrm{mn}} \Phi$

By substituting the expression for $\bar{\sigma}_{\mathrm{m}}$, defined by Equation (27) and recalling the definition of $\Phi$, the following general equation is obtained:
$A^{\prime}=m_{0} \int_{0}^{\infty} \sigma_{\operatorname{mn}}(\mathbb{E}) \varphi(E) d E$

Theoreticelly, $A^{\prime} / m_{0}$, defined by Equation (31), should be equal to the Corrected Reaction Rate (CRR) obtained from the activity measured of the irradiated foils.
B. Determination of Reaction Rates

Generally, the irradiation of activation detectors involves the exposure of a set of foils for an irradiation period $t_{i}$ in the neutron environment of interest. Following a waiting yeriod of $\dot{t}_{w}$, each foil is analyzed for specific Gamma-ray photo peaks and counted for a counting period of $t_{c}$. The net area under the photopeaks of interest is computer-calculated.

The result will be CPA, the Corrected Peak Area, and is related to CRR, the Corrected Reaction Rate, through the following equations:

$$
\begin{align*}
C P R R=A^{\prime} / m_{0}= & \lambda \frac{C P A}{\epsilon \vec{b}} \times \frac{1}{m^{\prime} p \hat{I}} \times \frac{\exp \left(\lambda t_{W}\right)}{1-\exp \left(-\lambda t_{i}\right)} \times \\
& \frac{C_{1} C_{2} C_{3} C_{4} C_{5}}{i-\exp \left(-\lambda t_{c}\right)} \tag{32}
\end{align*}
$$

where
$\leftarrow=$ detector efficiency
$\mathrm{b}=$ Gamma-ray branching ratio
$p=$ purity of the foil
$f=$ fraction of the isotope of interest ( $m_{0}=m^{\prime} p f$ )
$C_{0}=$ isomeric reaction correction factor
$C_{1}=$ reaction product burn up correction factor
$C_{2}=$ power correction factor
$C_{3}=$ self-shielding
$C_{4}=$ flux gradients
$C_{5}=$ other correction factors

## 1. Isomeric reaction correction

For such reactions as ${ }^{58} \mathrm{Ni}(\mathrm{n}, \mathrm{p}){ }^{58} \mathrm{Co}$, the activation product isotope, in this case ${ }^{58} \mathrm{Co}$, is the result of two different reactions, as shown in the following scheme:


The notations used are:

$$
\begin{aligned}
& \sigma_{\gamma} \quad=58_{\mathrm{Ni}} \text { thermal absorption cross section } \\
& \bar{\sigma}_{\mathrm{m}} \quad={ }^{58_{\mathrm{Ni}}} \text { average fast neutron cross section for the } \\
& \text { formation of }{ }^{58} \mathrm{~m}_{\mathrm{Co}} \\
& \overline{\mathrm{G}} \quad=5_{\mathrm{Ni}} \text { average fast neutron cross section for the } \\
& \text { formation of }{ }^{58} \text { Co } \\
& \left(\bar{\sigma}+\bar{\sigma}_{m}\right)={ }^{58}{ }_{N i} \text { total average fast neutron cross section for } \\
& \text { the formation of }{ }^{58} \mathrm{~m}_{\mathrm{Co}} \\
& \sigma_{\gamma}^{\mathrm{m}}={ }^{58 \mathrm{~m}_{\mathrm{Co}}} \text { thermal absorption cross section } \\
& \sigma_{\gamma^{\prime}}={ }^{58} \text { Co thermal absorption cross section } \\
& n(x) \quad=\text { number of atoms of nuclide } x \text { at time } t \\
& n_{0}(x)=\text { number of atoms of nuclide } x \text { at time } t_{0} \\
& n_{t w}(x)=\text { number of atoms of nuclide } x \text { after a waiting time } \\
& \text { tw. }
\end{aligned}
$$

During irradiation, the time rate of change of the target atoms ${ }^{58}{ }_{\mathrm{Ni}},{ }^{58 \mathrm{~m}} \mathrm{Co}$, and ${ }^{58} \mathrm{Co}$ are as follows;

$$
\begin{equation*}
\frac{\operatorname{dn}\left(58_{N i}\right)}{d t}=-n\left(58_{N i}\right)\left[\sigma_{0}^{E_{\text {thrs }}} \varphi(E) d E+\left(\bar{\sigma}+\bar{\sigma}_{m}\right) \int_{E_{\text {thrs }}}^{\infty} \varphi(E) d E\right] \tag{33}
\end{equation*}
$$

$$
\begin{align*}
\frac{d n\left({ }^{58} \mathrm{CO}\right)}{d t}= & n\left({ }^{58} \mathrm{Ni}\right) \bar{\sigma} \int_{E_{\text {thrs }}}^{\infty} \varphi(\mathbb{E}) d \mathrm{E}+\mathrm{n}\left({ }^{\left.58 \mathrm{~m}_{\mathrm{Co}}\right) \lambda_{\mathrm{m}}}\right. \\
& -\mathrm{n}\left({ }^{58} \mathrm{CO}\right)\left[\lambda+\sigma_{\gamma^{\prime}} \int_{0}^{E_{\mathrm{th}}} \varphi(\mathrm{E}) d \mathrm{E}\right] \tag{35}
\end{align*}
$$

Introducing the following abbreviations:

$$
\begin{align*}
\Phi_{t h} & =\int_{0}^{E_{\text {thrs }}} \varphi(E) d E \\
\Phi_{\mathrm{r}} & =\int_{E_{\text {thrs }}}^{\infty} \varphi(E) d E \\
E_{1} & =\sigma_{\gamma} \Phi_{t h}+\left(\bar{\sigma}+\bar{\sigma}_{m}\right) \Phi_{f}  \tag{36}\\
E_{2}= & \lambda_{m}+\sigma_{\gamma}^{m} \Phi_{t h}  \tag{37}\\
E_{3}= & =\lambda+\tilde{\sigma}_{r} \Phi_{t h} \tag{38}
\end{align*}
$$

Equations (33), (34), and (35) reduce to
$\frac{d n\left(58_{\mathrm{Ni}}\right)}{d t}=-E_{1} n\left({ }^{58_{\mathrm{Ni}}}\right)$

$$
\begin{aligned}
& \frac{d n\left(58 m_{\mathrm{Co}}\right)}{d t}=n\left(8_{\mathrm{Ni}}\right) \bar{\sigma}_{\mathrm{m}} \Phi_{\mathrm{f}}-\mathrm{E}_{2} n\left({ }^{58 \mathrm{~m}_{\mathrm{Co}}}\right) \\
& \frac{d n(58 \mathrm{Co})}{d t}=n\left(8_{\mathrm{Ni}}\right) \Phi_{\mathrm{f}} \bar{\sigma}+\lambda_{\mathrm{m}} n\left({ }^{\left.58 \mathrm{~m}_{\mathrm{Co}}\right)}-\mathrm{E}_{3} m\left({ }^{58} \mathrm{Co}\right)\right.
\end{aligned}
$$

with the following solutions:

$$
\begin{align*}
n\left(588_{N i}\right)= & n_{0}\left(58_{N i}\right) \exp \left(-E_{1} t_{i}\right)  \tag{39}\\
n\left({ }^{58 m_{C o}}\right)= & n_{0}\left({ }^{58 m_{C o}}\right) \exp \left(-E_{2} t_{i}\right)+n_{0}\left(58_{N i}\right) \Phi_{f} \\
& {\overline{\sigma_{m}}}_{E_{2}-E_{1}}=\exp \left(-\mathbb{E}_{1} t_{i}\right)-\exp \left(-\mathbb{E}_{2} t_{i}\right) \tag{40}
\end{align*}
$$

$$
n\left({ }^{58} C 0\right)=n_{0}(58 C 0) \exp \left(-E_{3} t_{i}\right)+\lambda_{m} n_{0}\left({ }^{58 m_{C 0}}\right) \times
$$

$$
\left[\frac{\exp \left(-E_{3} t_{i}\right)-\exp \left(-E_{2} t_{i}\right)}{E_{2}-E_{3}}\right]+n_{0}\left(58_{\underline{N} i}\right) \times
$$

$$
\Phi_{f}\left\{\left[\bar{\sigma}+\frac{\lambda_{m} \bar{\sigma}_{m}}{E_{2}-E_{1}}\right]\left[\frac{\exp \left(-E_{1} t_{i}\right)-\exp \left(-E_{3} t_{i}\right)}{E_{3}-E_{1}}\right]\right.
$$

$$
\begin{equation*}
\left.-\left[\lambda_{\mathrm{m}} \vec{\sigma}_{\mathrm{m}} /\left(E_{2}-E_{1}\right)\right]\left[\frac{\exp \left(-E_{3} t_{i}\right)-\exp \left(-E_{2} t_{i}\right)}{E_{2}-E_{3}}\right]\right\} \tag{41}
\end{equation*}
$$

During the waiting time, the time rate of change of ${ }^{58} \mathrm{~m}_{\mathrm{Co}}$ and ${ }^{58}$ Co is

$$
\begin{align*}
& \frac{d n\left(58 m_{C O}\right)}{d t}=-n\left(5^{58 m^{C O}}\right) \lambda_{m}  \tag{42}\\
& \frac{d n\left({ }^{58} \mathrm{CO}\right)}{d t}=n\left({ }^{58 \mathrm{~m}_{\mathrm{CO}}}\right) \lambda_{m}-\lambda_{n}\left({ }^{58} \mathrm{Co}\right) \tag{43}
\end{align*}
$$

with the following solutions:

$$
\begin{align*}
n_{t_{w}}\left({ }^{58 m_{C O}}\right)= & n_{0}\left({ }^{58 m_{C O}}\right) \exp \left(-E_{2} t_{i}\right) \exp \left(-\lambda_{m} t_{w}\right) \\
& +n_{0}\left({ }^{58}{ }_{N i}\right) \Phi_{f} \bar{\sigma}_{m}\left[\frac{\exp \left(-E_{1} t_{i}\right)-\exp \left(-E_{2} t_{i}\right) \exp \left(-\lambda_{m} t_{w}\right)}{E_{2}-E_{1}}\right] \tag{43}
\end{align*}
$$

Substituting Equation (43) in Equation (42), $n\left({ }^{58} \mathrm{Co}\right.$ ), the mumber of ${ }^{58}$ Co atoms after a waiting time of $t_{w}$ following an irradiation period of $t_{i}$ is given by

$$
n_{t_{w}}\left({ }^{58} \mathrm{Co}\right)=n\left({ }^{58} \mathrm{Co}\right) \exp \left(-\lambda t_{w}\right)+n\left({ }^{58 \text { iii }} \mathrm{Co}\right) \lambda_{m} \times
$$

$$
\begin{aligned}
& x\left[\frac{\exp \left(-\lambda t_{w}\right)-\exp \left(-\lambda_{m} t_{w}\right)}{\lambda_{m}-\lambda}\right]= \\
& =n_{0}(58 \mathrm{Co}) \exp \left(-E_{3} t_{i}\right)+\lambda_{m} n_{0}\left(58 m_{C o}\right)_{x} \\
& {\left[\frac{\exp \left(-E_{3} t_{i}\right)-\exp \left(-E_{2} t_{i}\right)}{E_{2}-E_{3}}\right]+n_{0}\left(58_{N i}\right) \Phi_{f}\{\bar{\sigma}+}
\end{aligned}
$$

$$
\begin{align*}
& +\frac{\lambda_{m} \bar{\sigma}_{m}}{E_{2}-E_{3}}\left[\frac{\exp \left(-E_{1} t_{i}\right)-\exp \left(-E_{3} t_{i}\right)}{E_{3}-E_{1}}\right]- \\
& \left.-\frac{\lambda_{m} \bar{\sigma}_{m}}{E_{2}-E_{1}}\left[\frac{\exp \left(-E_{3} t_{\dot{i}}\right)-\exp \left(-E_{2} t_{i}\right)}{E_{2}-E_{3}}\right]\right\} \exp \left(-\lambda t_{w}\right) \\
& +\lambda_{m}\left\{n_{0}\left({ }^{58 m_{C o}}\right) \exp \left(-E_{2} t_{i}\right)+\bar{\sigma}_{m} \Phi_{f} n_{0}\left(58_{N i}\right) \times\right. \\
& \left.\left[\frac{\exp \left(-E_{1} t_{i}\right)-\exp \left(-E_{2} t_{i}\right)}{E_{2}-E_{1}}\right]\right\}\left[\frac{\exp \left(-\lambda t_{w}\right)-\exp \left(-\lambda_{m} t_{w}\right)}{\lambda_{m}-\lambda}\right] \tag{44}
\end{align*}
$$

Equation (44) can be simplified by considering the following nuclear data:

$$
\begin{aligned}
t / 2\left({ }^{58} \mathrm{Co}\right) & =71.23 \mathrm{~d} \\
t / 2(58 \mathrm{~m} \mathrm{Co}) & =9.0 \mathrm{hr} \\
\sigma_{\gamma} & =4.4 \mathrm{barn} \\
\bar{\sigma} & =74 \mathrm{mbarn} \\
\bar{\sigma}_{\mathrm{m}} & =28 \mathrm{mbarn} \\
\sigma_{\gamma}^{\prime} & =2.5 \times 10^{3} \mathrm{barn} \\
\sigma_{\gamma}^{m} & =1.4 \times 10^{5} \mathrm{barn}
\end{aligned}
$$

If $\Phi_{\text {th }}$ and $\Phi_{\mathrm{f}}$ are assumed to be:
$\Phi_{\text {th }}=\Phi_{\mathrm{f}}=10^{14} \mathrm{n} / \mathrm{cm}^{2} \mathrm{sec}$
and $t_{i}<100 \mathrm{hr}$, then
$\begin{aligned} E_{1} & =4.50 \times 10^{-10} \quad \mathrm{sec}^{-1} \\ \lambda & =1.13 \times 10^{-7} \quad \mathrm{sec}^{-1}\end{aligned}$
$\lambda_{m}=2.14 \times 10^{-5} \mathrm{sec}^{-1}$
$\exp \left(-E_{1} t_{i}\right) \cong 1$
Furthermore, if:
$n_{0}\left({ }^{58} \mathrm{Co}\right)=n_{0}\left({ }^{58 m_{C O}}\right)=0$, and $t_{w}>2 d$
Equation (44) reduces to

$$
\begin{align*}
n_{t_{w}}\left({ }^{58} \mathrm{Co}\right)= & n_{0}\left(58_{N i}\right) \Phi_{f} \exp \left(-\lambda t_{w}\right)\left\{\bar{\sigma}+\frac{\sigma_{m} \lambda_{m}}{E_{2}}\right. \\
& {\left[1-\exp \left(-E_{3} t_{i}\right)\right] / E_{3}+\bar{\sigma}_{m}^{\prime} \frac{\Phi_{t h} \sigma_{r} m}{E_{2}} } \\
& {\left.\left[\frac{1-\exp \left(-E_{2} t_{i}\right)}{\mathbb{E}_{2}}\right]\right\} } \tag{45}
\end{align*}
$$

## 2. Reaction product burn-up

This correction factor applies to such reactions as $5^{\mathrm{Ni}(\mathrm{n}, \mathrm{p})}{ }^{58}$ Co where the reaction product isotope has a high capture cross section. In this case, the reaction product, ${ }^{58}$ Co, has a capture cross section of $2.5 \times 10^{3}$ barn, while $5^{5 \mathrm{~m}}$ Co has a capture cross section of $1.4 \times 10^{5}$ barn. This means that during the irradiation time $t_{i}$, part of ${ }^{58}$ Co atoms are subject to "burn-up" by thermal neutrons. If $C_{1}$ denotes
the burn-up correction factor and $n^{\prime} t_{T V}\left({ }^{58} \mathrm{Co}\right)$ the number of ${ }^{53}$ Co atoms with no burn-up, is given by Bquation (45). Substituting Equations (47) and (45) in Bquation (46) the burn-up correction becomes:

$$
\begin{aligned}
C_{1}= & \left(\bar{G}+\bar{\sigma}_{m}\right)[1-\exp (-\lambda t)] / \lambda \\
& \left\{\left(\bar{G}+\frac{\bar{\sigma}_{m} \lambda_{m}}{\bar{U}_{2}}\right)[1-\exp (-\lambda)] / \lambda+\right. \\
& \left.+\left(\bar{G}_{m} \sigma_{\gamma}^{m} \Phi{ }_{t h} / E_{2}\right)\left[1-\exp \left(-E_{2} t\right)\right] / E_{2}\right\}^{-1}
\end{aligned}
$$

## 3. Power correction

This factor is used to determine any possible reactor power fluctuation during the irradiation time. For this purpose, each set of activation foils intended for a given irradiation included an Au-Al wire After the irradiation, the wire activity, $A_{x}$, is measured and compared with the "reference" activity, $A_{\text {ref }}$. The latter is the activity of an identical An-Al wire irradiated in a constant flux corresponding to $5 \mathrm{~m} . \quad$ Both $A_{x}$ and $A_{\text {ref }}$ are expressed in terms of the net area under the 0.412 MeV photopeak of ${ }^{198} \mathrm{Au}$, corrected for $t_{i}$ end $t_{\mathrm{t}: \text {. }}$ The power correction factor $C_{2}$ is defined by $c_{2}=\frac{A_{\text {ref }}}{A_{x}}$

As the celibration wires used with each foil set had the same

Weight and dimension of the "reference" wire and were counted under identical conditions, no efficiency correction was needed.

## 4. Deiector efficiency

Detector efficiency as a function of energy was determined for the $G e(L i)$ detector by comparing known emission rates with the measured counting rates for gama radiations of various energies provided by absolute standards. The principal standards used in this experiment were ${ }^{133} \mathrm{Ba},{ }^{22} \mathrm{Na}$, ${ }^{137} \mathrm{Cs}$, and ${ }^{60} \mathrm{Co}$, of very similar geometry. Each standard consisted of the actual radiation source with a diameter of apo proximately $1 / 8^{\prime \prime}$ thick deposited at the center of $1^{\prime \prime}$ disks. The activity of the standards was calculated on the basis of the following gamma peaks:
$133 \mathrm{Ba}: 0.276,0.302,0.356$, and 0.382 MeV
27 Na : 0.511 , and 1.275 MeV
${ }^{137} \mathrm{Cs}: 0.622 \mathrm{MeV}$
$6^{60} \mathrm{Co}$ 1.173, and 1.332 MeV
covering an energy range of 0.23 MeV to 1.33 MeV . The detection efficiency vs. gama enersy, plotted on a log-los basis, Was found to be a straight lire, consequently, the gamma efficiencies at energies greater than 1.33 MeV could be obtained by extrapolation. In oraer to verify the validity of the extrapolation, used primerily to obtain the 1.595 MeV gama detec-
tion efficiency of ${ }^{140} \mathrm{La}$, a ${ }^{207}$ Bi standard was used. As this standard had a slightly different geometry, it was intercalibrated with the other four standards. ${ }^{207}$ Bi with its gamma energies of $0.570,1.063$, and 1.771 MeV extended the range of measured detection efficiency to 1.771 MeV .

Efficiency measurements were carried out for several assembly shelves. In each case the natural logarithm of the detection efficiency vs. the natural logarithm of the photopeak energies was plotted. The results for each shelf yielded straight lines with identical slopes.

## 5. Other corrections

Equations (40) show other possible correction factors that may need to be determined for specific foils and/or particular counting condition
$C_{5}=\frac{\Lambda_{1}}{\Lambda_{2} \Lambda_{3}}$
where
$\Lambda_{1}=$ correction factor for variations in foil diameter $\Lambda_{2}=$ correction for gamma self-shielding $\Lambda_{3}=$ foil weight normalization factor
$\dot{n}_{1}$ had to be accounted for in a limited number of cases where the foil diameters were appreciably different than the diameter of the absolute standards. This factor was defined as
$\Lambda_{1}=\frac{A\left(d=1 / 8^{\prime \prime}\right)}{A\left(d \neq 1 / 8^{\prime \prime}\right)}$
where $A\left(d=1 / 8^{\prime \prime}\right)$ is the net area under the photopeak of a foil with the standard size, and $A(d \neq 1 / 8 \prime \prime)$ indicates the net photopeak area of a foil with a different diameter.

To determine $\Lambda_{1}$ experimentally, two foils of identical properties but different diameters (one with $d=1 / 8^{\prime \prime}$ ) were irradiated together for a specific $t_{i}$. The net area under the principal photopeak of each foil was corrected for the foil mass, $t_{w}$ and $t_{c}$.
$\Lambda_{2}$, negligible for nearly all foils (except In), is defined by the following approximation (40):

$$
\begin{equation*}
\lambda_{2}=1-\exp (-\mu x) / \mu x \tag{47}
\end{equation*}
$$

where $\mu$ is the mass absorption coefficient of the foil $\left(\mathrm{cm}^{-1}\right)$ and $x$ is the thickness of the foil. For In and $A u$ foils Equation (47) was substituted for by Bothe's approximation (41) defined as
$\Lambda_{2}=1-\Sigma_{a} x\left(1-\log \Sigma_{a} x\right)$
$\Lambda_{3}$ was used for the foils which were weighed at the $A L R R$. Since the majority of the foils were obtained and weighed with high precision at ANL, the weight of foils weighed at ALRR was normalized to ANTL values. This was accomplished by weighing several foils at both ANL and ALRR. The ratios of ALRR weight to ANL weight, $W_{i}(A L R R) / W_{i}(A N L)$, were plotted vs. the $A L R R$ weights $W_{i}(A L R R)$ and a best fit straight line drawn. The value of $\Lambda_{3}$ for each foil weighed at $A L R R$ was selected to
be the ratio $W_{i}(A L R R) / W_{i}(A N L)$ corresponding to $W_{i}$ (ALRR). For the special case of $238_{\mathrm{U}}$ foils en additional correction factor was considered. This correction factor, $\Omega$, was to take into account the presence oi $0.04 \% 235 \mathrm{U}$ in the ${ }^{238} \mathrm{U}$ samples. $\Omega$, defined as the ratio of the fission rate of pure ${ }^{238} \mathrm{U}$ to the fission rate of the ${ }^{238} \mathrm{U}_{\mathrm{U}}$ sample, is given by
$\Omega=\left[1+4 \times 10^{-4}\left(\bar{\sigma}_{25}^{f} / \bar{\sigma}_{28}^{f}\right)\left(\bar{\Phi}_{C \alpha} / \bar{\Phi}_{\mathbf{f}}\right)\right]^{-1}$
where $\bar{\Phi}_{C \alpha}$ and $\bar{\Phi}_{f}$ indicate neutron flux above the cadmium cuioff, and the flux above the $238_{\mathrm{U}}(\mathrm{n}, \mathrm{f})$ reaction threshold, respectively. Using several conservative estimates of $\bar{\Phi}_{C d} / \bar{\Phi}_{\mathrm{I}}$, it was determined that the value of $\Omega$ was very close to 1 and thus, no correction was needed.
V. EXPeRIMENT

The purpose of the experiment was to masure the energy distribution of neutrons in certain ALRR experimental facilities.

Neutron induced reactions with various nuclides were studied and a number were selected which satisfied the dual recuirement of having adequate detector response in the energy range of $0.1 \mathrm{MeV} \leqslant E \leqslant 10 \mathrm{MeV}$ and leading to detectable photon emission.

Packets of activation foils were assembled, observing necessary precaution to prevent cross-contamination, high exposure to thermal neutrons, and physical damage. Packets were inserted into the irradiation region of interest and exposed to the neutron flux for an appropriate period of time.

The activated foils were taken from the reactor, the packets were disassembled, and each foil was allowed to decay for an appropriate time until a decay rate was reached which permitted accurate measurement. Each foil activity was measured v:sing conventional detector and multichannel analyzer procecures.

The experiment was analyzed for departure irom ideal or theoretical conditions, appropriate correction factors were devised, end necessary numerical corrections determined. The aporatus was calibrated, and routine checks were conducted to ascertain thet consistent performance was maintained.

## A. Irradiation Facilities

All irradiations were conducted in the ines Lebonstory Research Reactor (ALRR)*. The ALRR was a heterogenous, heavy water reactor with an operating power level of 5 MW . The fully enriched core was moderated, cooled, and reflected by heavy water.

The core cortained 24 fuel eiements of the paraliel plate type. Each element was $3^{\prime \prime}$ x. $3^{\prime \prime}$ in cross section and 521" long overall with the fueled region being 24 5/8" long.

A total of 35 experimentel facilities penetrated the shielding to permit access to core radiation. The facilities selected for this work were the R-3 (a rabbit facility) and the V-1 (vertical thimble). The locations and dimensions of these facilities are illustrated in Appendix A.

## B. Counting Equipment

The photon emissions from irradiated foils were analyzed. according to their energies using an ORTEC WIN series coaxial Ge(Ii) detector with the following peripheral equipment:

- ORTEC model 120-4 preamplifier
- Canberra model 1417B amplifier
- Canberra model 1400 Nim Bin

[^4]- ITucleer Data 50/50 analyzer system - 4096 channels PDP-8/I minicomputer
- Perinheral Equipment 9-track tape drive
- $15^{\prime \prime} \times 16^{\prime \prime} \times 201$ inside dimensions lead cave with $2 \prime 1$ thick lead and an inner lining of cadmium and copper
- Raciaition Instrument Developnent Laboratory pulser model 47-2
- Serviey Nucleonics tail pulse generator model RP-1.

Foils were placed in the lead cave in an aluminum holder which iixed the distance between the foils and the Ge(Li) detector at a value between 10 and 30 cm . The system's live time was checked by the pulser. The area under each photoelectric peak of interest was determined with the aid of the ICDEAX computer program*. ICPEAX fits an experimentally determined photoelectric peak with a gaussian curve, determines the background-corrected area under the gaussian peak, and finds the energy to be associated with the peak (42).
C. Choice of Detectors

The criteria for the selection of specific nuclides to Ce empicyed as ihreshold detector were based upon such require. ments as terget materiei characteristics (availability, puri-

[^5]ty, etc.); reaction data (type, ma.gnitude and knowledge of cross section, $\in t c$. ) ; and the product isotope nuclear data (decay scheme, half life, etc.). The above criteria are listed as follows:

- The available material purity should be high enough that interferring impurity reactions can be neglected.
- The material should be chemically stable and capable of beins formed into thin (few mils) foils.
- The reaction should have a reasonably well known cross section.
- The reaction should have adequate "sensitivity" and a high enough yield to provide good counting statistics.
- The product isotope must be capable of being gamma counted, it should have an adequate half life (more than 10 minutes), and a well established decay scheme.

Based upon the above considerations, an optimum set of threshold detectors was seiected to yield the reactions listed in Table 2. In addition to the relatively high energy detector reactions listed in Table 2 , the ( $n, \gamma$ ) reactions listed in Table 3 were used to measure lower energy neutrons. The uncerieinty associated with $\sigma(\Xi)$ of the above reactions are listea in Table 6 (43, 44).

Table 2. Threshold reaction data

| Reaction | Reaction product helf life | Gamma energy (KeV) | Gemma yield \% | Taraet purity $\%$ |
| :---: | :---: | :---: | :---: | :---: |
| $24 \mathrm{Mg}(\mathrm{n}, \mathrm{p})^{24} \mathrm{Na}$ | 15.00 hr | 1368.60 | 99.993 | 99.8 |
| $27 \mathrm{Al}(\mathrm{n}, \alpha)^{24} \mathrm{Na}$ | 15.00 hr | 1368.60 | 99.993 | 99.99 |
| ${ }^{27} \mathrm{Al}(\mathrm{n}, \mathrm{p})^{27} \mathrm{Mg}$ | 9.46 min | $E_{1}=843.73$ $E_{2}=1014.44$ | $\begin{aligned} & y_{1}=71.4 \\ & y_{2}=28.6 \end{aligned}$ | 99.99 |
| ${ }^{46} \mathrm{Ti}(\mathrm{n}, \mathrm{p})^{46} \mathrm{Sc}$ | 83.85 d | $\begin{aligned} & \mathbb{E}_{1}=889.258 \\ & E_{2}=1120.516 \end{aligned}$ | $\begin{aligned} & y_{1}=99.984 \\ & y_{2}=99.987 \end{aligned}$ | 89 |
| $4^{7 i}(\mathrm{n}, \mathrm{p})^{47} \mathrm{Sc}$ | 3.39 d | 159.39 | 69.0 | 99 |
| ${ }^{48} \mathrm{Ti}(n, p){ }^{48} \mathrm{Sc}$ | 43.8 hr | $\begin{aligned} & E_{1}=983.4 \\ & E_{2}=1 C 37.4 \\ & E_{3}=131 i .8 \end{aligned}$ | $\begin{aligned} & y_{1}=99.987 \\ & y_{2}=97.5 \\ & y_{3}=99.992 \end{aligned}$ | 99 |
| ${ }^{54} \mathrm{Fe}(\mathrm{n}, \mathrm{p})^{54 \mathrm{Mn}}$ | 312.6 d | 834.827 | 99.97 | 99.9 |
| ${ }^{56}{ }_{\mathrm{Fe}}(\mathrm{n}, \mathrm{p})^{56} \mathrm{Mn}$ | 2.576 hr | 846.9 | 99 | 99.9 |
| $58_{\text {Ni }(n, p)}{ }^{58} \mathrm{Co}$ | 71.23 d | 810.757 | 99.44 | 99.99 |
| ${ }^{63} \mathrm{Cu}(\mathrm{n}, \alpha){ }^{60} \mathrm{Co}$ | 5.268 y | $\begin{aligned} & E_{1}=1173.208 \\ & E_{2}=1332.464 \end{aligned}$ | $\begin{aligned} & y_{1}=99.86 \\ & y_{2}=99.986 \end{aligned}$ | 99.9 |
| ${ }^{90} \mathrm{Zr}(\mathrm{n}, 2 \mathrm{n})^{89} \mathrm{Zr}$ | 78.4 hr | 910 | 99 | 99.942 |
| ${ }^{115} \mathrm{In}\left(\mathrm{n}, \mathrm{n}^{1}\right)^{115} \mathrm{In}^{\mathrm{m}}$ | 4.50 hr | 335.2 | 47 | 99.99 |
| ${ }^{232} \mathrm{Th}(\mathrm{n}, \mathrm{f}){ }^{140} \mathrm{La}$ | 40.26 hr | 1596.18 | 95.33 | 99.8 |
| ${ }^{233}{ }_{U}\left(\mathrm{n}, \mathrm{f}^{1}\right)^{140} \mathrm{La}$ | 40.26 hr | 1590.18 | 95.33 | 99.95 |

Table 3. Activation reactions deta

| Reaction | Reaction mroduct helf-life | Gamna energy ( KeV ) | Gamina <br> yield <br> (\%) | Torget purity (\%) |
| :---: | :---: | :---: | :---: | :---: |
| ${ }^{58} \mathrm{Fe}(\mathrm{n}, \gamma){ }^{59} \mathrm{Fe}$ | 44.6 d | $\begin{aligned} & E_{1}=1099.224 \\ & E_{2}=1291.564 \end{aligned}$ | $\begin{aligned} & \mathrm{y}_{1}=55.5 \\ & \mathrm{y}_{2}=44.1 \end{aligned}$ | 99.9 |
| ${ }^{59} \mathrm{Co}(\mathrm{n}, \gamma)^{60} \mathrm{Co} *$ | 5.268 d | $\begin{aligned} & E_{1}=1173.208 \\ & E_{2}=1332.464 \end{aligned}$ | $\begin{aligned} & y_{1}=99.86 \\ & y_{2}=99.986 \end{aligned}$ | 99.9 |
| ${ }^{63} \mathrm{Cau}\left(\mathrm{n}, 8\right.$ ) ${ }^{64} \mathrm{Cu}$ | 12.701 hr | 511.002 | 36.8 | 99.9 |
| ${ }^{197} \mathrm{Au}(\mathrm{n}, \mathrm{r})^{198} \mathrm{Cu**}$ | 2.698 d | 411.794 | 95.48 | 99.999 |
| * 0.1\% CO-A1 | Al: 99.999\%; | Co: 99. |  |  |
| ** $0.135 \%$ Au- 41 | Au: 99.999\%; | Al: 99. |  |  |

## D. Irradiation of Detectors

High purity activation detectors, provided by $A I R R$ and Argone Nationel Laboratory, vere prepared in the form of $1 / 8^{\prime \prime}$ diameter and 0.5 to 5 mils thick foils. Each set of foils, to be irradiated simultaneously, vas racked, cadrium covered, and cortained in an aluminum or polyethylene rabbit, the choice depending on the duration of irradiation.

Aluminum spacers, approximately 10 mils thick, were used to separate the foils. 238 and ${ }^{232_{\mathrm{Th}}}$ foils were inoividuelly wrapped in 5 mils thick aluminum to contain possible recoil fission fragments and to prevent cross-contamination between foils.

Comparisan of experimental results involving the irradiation of single foils ( 1 foil with no aluminum spacer) and sets of activation foils indicated that, within experimental errors, the perturbation effects on the fast neutron flux are negligible due to the presence of multiple foil in a set. Furthermore, irradiation of foils with diameters as large as $1.5^{\prime \prime}$ indicated that the neutron flux at the center of the rabbit tube (where the cacmium-covered foil set is mounted) is uniform over the region occupied by the foils. Therefore, no corrections for flux perturbation due to the presence of multivle foil set or for the lack of uniformity were necessary. The duration of on irradation was dictated by the fol10: ine:

- The activity of the isotope of interest should be high enough to provide good counting statistics.
- The isotope activity should not be great enough to cause a. dead time problem with the counting ecuipment.
- The irradiation time should be less tran the effective inreactor half life of the foil, $T 1 / 2(e f \hat{I})$, defined below, to prevent saturation activity problems.

In order to determine $T 1 / 2(e f f)$, tine production rate of an isctope can be written as
$\frac{d n}{d t}=N_{0} \sigma(E) \varphi(E)$
The concentration will be diminished by radioactive decay and the trensmutation of the product radionuclide
$-\frac{d n}{d t}=n\left[\lambda+\varphi(\mathbb{E}) \sigma^{\prime}(E)\right]=n \lambda_{e f f}$
where $\sigma^{\prime}(B)$ is the microscovic cross section of the resultant nuclide for iransmutation to still another nuclide. Saturation activity occurs when the rate of production equals the rate of Ioss:
$I_{0} \sigma(\mathbb{D}) \varphi(\vec{I})=n_{\text {equil }} \lambda_{\text {eff }}$
and the time constant that determines the approach to saturated Ectivity is
$T 1 / 2(e f f)=\ln 2 / \lambda_{e f f}=\ln 2 /\left[\lambda+\varphi(B) \sigma^{\prime}(E)\right]$

By using conservative estinates of $\varphi(\mathbb{E})$ and $G^{\prime}(\mathbb{E})$, approzimate values of T $1 / 2$ (eff) were determined. The irradiation time $t_{\text {I }}$ of each set of foils was selected to be less than the lowest T $1 / 2(\in \hat{i} f)$ of each set.

Based upon these considerations, the foil sets were irradiated as indicated below:

- Short irradiations, with $t_{i}<30$ minutes, were used for foil sets conteining ${ }^{27} \mathrm{AI},{ }^{115} \mathrm{In},{ }^{197} \mathrm{Au},{ }^{56}{ }_{\mathrm{Fe}},{ }^{238}{ }_{\mathrm{U}},{ }^{59} \mathrm{Co}$, and ${ }^{63}$ Cu isotopes.
- Iong irradiations, with $t_{i} \leqslant 15$ hours, were used for foil sets conteining ${ }^{63} \mathrm{Cu}$ (for $\mathrm{n}, \alpha$ reaction), ${ }^{90} \mathrm{Zr}$, and ${ }^{54} \mathrm{Fe}$.
- Intermediate irradiations, with $t_{i} \leqslant 5$ hours, were used for foil sets containing the other foils.

Each foil set contained at least two reference monitors, Au-Al rire and $\mathbb{N i}$ foil, for the purpose of run-to-run power Ievel normelication.

## E. Counting

The half life of the isotope of interest, the helf lives of interferring isctones, and the actirity levels of the various nuclides at the ence of tine irradiation dictated the waiting time $t_{\text {in }}$. A tine time when the activities were measured, the counting raies had to be withir the limits imposed by good countine statistics and the dead time of the countine equiment. To establish the georetry to be used wen the Ge(fi) count-
ing efificiency was detemined, each foil was placed at the center of a "dummy source." The "dummy source" consisted of a plastic disk having the same dimensions and material properties of the absolute standards used for the detector efficiency determinations. The "dummy source" containing the sample was placed on the same aluninum planchet used for the absolute standards and courted.

Bach semple vas counted at least three times within a time period of $3-4$ half lives after the end of the irradiation. If the three counts, corrected for waiting and counting time, did not agree to within $\div 4 \%$, additional counts were taken. Furthermore: in cases with possible interferring activities, the source of the counts was checked by cetermining the actual half life of the activity being measured and comparing this value with the published half life of the nuclide of interest.

The principal gama ray of each radionuclide vas counted for a sufficiently long period of time to provide counting statistics uncertainty of less than $2 \%$. The source-to-detector distance was adjusted to provide maximum count rate without exceeding $15 \%$ analyzer dead time. The typical source-to-detector distance was 20 cm .

## VI. RESULTS AMD DISCUSSIONS

Preliminary analysis of the experimental data indicated a high degree of inconsistency between ${ }^{27} \mathrm{Al}(\mathrm{n}, \mathrm{p}){ }^{27} \mathrm{Mg}$ reaction rates and the reaction rates determined for other foils used in this work. Such inconsistencies were attributed to the experimental limitations associated with the relatively short helf life ( 9046 minutes) of ${ }^{27} \mathrm{Mg}$. Since the ${ }^{27} \mathrm{Mg}$ activity had to be measured soon after the end of irradiation, the exposure of aluminium foils were kept at a minimal level. A short irradiation time, resuiting in low induced activity, made the safe handing of the samples possible with no needed recourse to the use of the hot cell facility.

Generally, handing the irradiated foils consisted of opening the rabbit, removing the cadmium cover, identifying the foils, and finally preparing the foils for analysis. This process required a cooling period ranging from a few hours to several days, depending on the exposure, type, and number of foils. In order to reduce the ${ }^{27} \mathrm{Mg}_{\mathrm{Mg}}$ cooling period to a few minutes, several measurements involving foils with irradiation times ranging from 0.5 to 4 minutes were taken. To further reduce the total activity of the foil set, the diameters of some selected Foils were reduced to half the stendard size.

The reaction rates obtained from these irradiation were inconsistent with each other and with the resuits obtained from the other foil sets involving longer irradiation times, The
deviation from the mean $\left(\bar{x}-x_{i}\right) *$ ranged from $|(18.89-12.1)|=$ $=6.79 \mathrm{cpm}$ for $t_{i}=2 \mathrm{~min}, t_{w}=75 \mathrm{~min}$, and $d=1 / 16^{\prime \prime}$; to $|(18.89-28.9)|=10.01 \mathrm{cpm}$ for $t_{i}=0.5 \mathrm{~min}, t_{w}=25 \mathrm{~min}$, and $d=1 / 8^{\prime \prime}$. The standard deviation was

$$
G_{\bar{x}}=\left[\frac{1}{7} \sum_{i=1}^{8}\left(\bar{x}-x_{i}\right)^{2}\right]^{1 / 2}=6.29 \mathrm{cpm}
$$

In order to make a qualitative determination of the experimental errors, it was assumed that the major sources of errors were: $\Delta t_{i}= \pm 5$ seconds in $t_{i}$ and $\Delta A=A^{1 / 2}$ in the measured activity A. For $t_{i}=0.5$ minute, $t_{w}=20$ minutes, and $A=152$ counts; $\frac{\sigma A_{0}}{A_{0}}$ was found to be

$$
\begin{gathered}
A_{0} / A_{0}=\sqrt{\left\{\exp (\lambda t w)\left(\Delta A / A_{0}\right) /[1-\exp (-\lambda t i)]\right\}^{2}+} \\
\cdots \\
\{\lambda \Delta t i \exp (-\lambda t i) /[1-\exp (-\lambda t i)]\}^{2}= \\
\\
=18.3 \%
\end{gathered}
$$

* $x_{i}$ is the reaction rate, in counts per minute, corrected for $t_{i}, t_{w}, t_{c}, m(A l)$, and $C_{2}$ (the power correction factor).

For $\dot{t}_{i}=1$ minute, $t_{w}=35$ minutes, and $A=123$ counts $/ 10$ minutes; the result was

$$
\sigma_{A_{0}} / A_{0}=12 \%
$$

Besed on the above considerations, it was concluded that:

- the errors introduced by $\Delta t_{i}(\div 5 \mathrm{sec})$ and $\pm \Delta \mathrm{A}$ were substantial, but could not account for the inconsistencies observed. in the reaction rates;
- $\Delta t_{i}$ could vary from irradiation to irradiation, and its velue could be higher than the assumed $\pm 5$ seconds (in fact, the power correction factors showed a fluctuation of 0.89 to 1.07); - at higher $t_{i}$ (where the relative contribution of $\Delta t_{i}$ was expected to be less) the reauction in the foil diemeter could have introduced errors in the value of detector efficiency. Eecause the inconsistencies in the uncertainties related to the ${ }^{27} \mathrm{AI}(n, p){ }^{27} \mathrm{Mg}$ reaction could not be understood, all the experimental data obtained for this reaction vere discarded. The deletion of ${ }^{27} \mathrm{AI}(\mathrm{n}, \mathrm{D})^{27} \mathrm{Mg}$ reaction would not alter the final unfolding solution appreciably. This is so because the ${ }^{27} \mathrm{Al}(\mathrm{n}, \mathrm{p})^{27} \mathrm{Fg}$ resporse function, covering an energy range from about 3 to 13 KeV , is rexlaced by the response function of the 46 Ti $(n, p)^{46}$ Sc reaction wich covers approximately the same energy range. However, within this energy range the cross section uncertainty of the ${ }^{4 \sigma_{T i}(n, D)}{ }^{i \sigma_{S C}}$ Sc reaction is higher then tiat for the elaminum reaction (see Table 6).


## A. SAND-II Runs

is mentioned in section IV, the oricinal SAIDDII code uses an energy range between $10^{-10}$ and 18 liev wich is civicied into 620 intervals. Thus, with a given input spectrum, the original code solves IV (number of foils used) linear activity equations for 621 unimowns. Jowever, given the limited number Of appropriate activation reactions and the smooth behavior of $\sigma(E)$ fcr gost threshold reactions of interest, the number of energy intervals can be reduced. Many laboratories (e. ©. int, RCiN-Netherlands, KFA-Germany, etc.) use modified SAHD-II codes with the energy range divided into a much smaller number of energy intervals. For instance, SAIVD-50 (Euroatom-Ispra, Italy) has only 50 energy subdivisions.

The code used in this work covers an energy range between $10^{-10}$ and 20 MeV , divided into 100 intervals. The first 10 intervals cover the range $10^{-10}$ to $5.5 \times 10^{-7} \mathrm{MeV}$, the next 60 intervals cover $5.5 \times 10^{-7}$ to 1.0 MeV , and the last 30 interVals cover the range from 1.0 to 20 MeV .

For an adequate choice $O$ input spectrum several spectra were tested (45). Included were:

$$
\begin{aligned}
& \varphi_{0}=1 / E, \text { and } \\
& \varphi_{0}=k E^{\alpha} \exp (-3 E / 2 \beta)+K \frac{1}{E^{\beta}}
\end{aligned}
$$

which were used with several values of $k, \alpha, \beta$, and $K$ (46). The spectra shown in Figures 1 and 2 were selected as the R-3


Figure 1. Input spectrum for the R-3 facility (flux in $n / \mathrm{cm}^{2}$ sec MeV )


Figure 2. Input spectrum for the $V-1$ facility (flux in $n / \mathrm{cm}^{2}$ sec MeV )
and $\mathrm{V}-1$ ALRR facilities input spectra for the SAND-II code. These spectra, $\Phi_{0}(R-3)$ and $\Phi_{0}(V-1)$, provided the final solution with the least number of iterations.
B. Solution Spectrum for the R-3 Facility

The solution spectrum for $R-3$ involved the use of $\varphi_{0}(R-3)$ as the input spectrum and the 17 foil reaction data, presented in Table 40 The final unfolding solution was reached after a total of 4 iterations ( 2 iterations prior to the deletion of ${ }^{48} \mathrm{Ti}$, and 2 iterations after) with $= \pm 3.36 \%$ as the deviation of measured from calculated activities (see Appendix B). In reaching the final solution, SAND-II discarded the ${ }^{48} \mathrm{Ti}(n, p){ }^{48}$ Sc reaction. In fact, following the first two iterations, the "percent deviation" of this reaction, defined as: [100 (measured activity/calculated activity - 1)] was the highest.

Figures 3 and 4 illustrate the initial solution (before the rejection of the ${ }^{48} \mathrm{Pi}(\hat{n}, \mathrm{p})^{48} \mathrm{Sc}$ ) and the innal solution (after the rejection of the ${ }^{48} \mathrm{Ti}(\mathrm{n}, \mathrm{p})^{48} \mathrm{Sc}$ reaction) over the entire energy range: $10^{-10}$ to 20 MeV . The SAND=II integral flux results indicated that the flux of fast neutron with $E \geqslant 1 \mathrm{MeV}$ in $\mathrm{R}-3$ was $1.12 \times 10^{13} \mathrm{n} / \mathrm{cm}^{2} \mathrm{sec}$. This figure is in good agreement with the ALRR measured value of $10^{13} \pm 25 \%^{*}$.

* Based on ${ }^{58} \mathrm{Ni}(\mathrm{n}, \mathrm{p}){ }^{58}$ Co reaction (private communication from Bruce Link of $A L R R)_{\text {。 }}$

Table 4. R-3 saturated measured activities and reactions response boundaries

| Reaction | $\begin{gathered} \text { measured } \\ \text { activity } \\ \text { (dps/nucleus) } \end{gathered}$ | reaction response boundaries ( MeV ) |  |
| :---: | :---: | :---: | :---: |
|  |  | lower | upper |
| ${ }^{27} \mathrm{Al}(\mathrm{n}, \alpha)^{24} \mathrm{Na}$ | $1.6 \times 10^{-14}$ | 6.7 | $1.7 \times 10^{1}$ |
| ${ }^{197} \mathrm{Au}(\mathrm{n}, \gamma){ }^{198} \mathrm{Au}$ | $3.6 \times 10^{-8}$ | $4.2 \times 10^{-6}$ | $4.5 \times 10^{-5}$ |
| ${ }^{59} \mathrm{Co}(\mathrm{n}, \gamma)^{60} \mathrm{Co}$ | $1.8 \times 10^{-9}$ | $8.4 \times 10^{-7}$ | $1.7 \times 10^{-4}$ |
| ${ }^{63} \mathrm{Cu}(\mathrm{n}, \gamma){ }^{64} \mathrm{Cu}$ | $1.3 \times 10^{-10}$ | $5.5 \times 10^{-7}$ | $1.9 \times 10^{-2}$ |
| ${ }^{58} \mathrm{Fe}(\mathrm{n}, \gamma){ }^{59 \mathrm{Fe}}$ | $4.2 \times 10^{-11}$ | $8.4 \times 10^{-7}$ | $3.6 \times 10^{-4}$ |
| ${ }^{54} \mathrm{Fe}(\mathrm{n}, \mathrm{p})^{54} \mathrm{Mn}$ | $1.1 \times 10^{-12}$ | 2.3 | 9.0 |
| ${ }^{115} \operatorname{In}(\mathrm{n}, \mathrm{n})^{115} \mathrm{In}^{\mathrm{m}}$ | $3.0 \times 10^{-12}$ | $6.6 \times 10^{-1}$ | 5.5 |
| ${ }^{58} \mathrm{Ni}(\mathrm{n}, \mathrm{p}){ }^{58} \mathrm{Co}$ | $1.4 \times 10^{-12}$ | 2.0 | 8.2 |
| ${ }^{90} \mathrm{Zr}(\mathrm{n}, 2 \mathrm{n}){ }^{89} \mathrm{Zr}$ | $4.2 \times 10^{-14}$ | $1.3 \times 10^{1}$ | $1.9 \times 10^{1}$ |
| ${ }^{46} \mathrm{Ti}(\mathrm{n}, \mathrm{p})^{46} \mathrm{Sc}$ | $1.4 \times 10^{-13}$ | 3.7 | $1.4 \times 10^{1}$ |
| $47_{\mathrm{Ti}}(\mathrm{n}, \mathrm{p})^{47} \mathrm{Sc}$ | $3.0 \times .10^{-13}$ | 1.8 | 8.2 |
| ${ }^{232} \mathrm{Th}(\mathrm{n}, \mathrm{f})^{140_{\mathrm{La}}}$ | $1.1 \times 10^{-12}$ | 1.4 | 8.2 |
| ${ }^{238} \mathrm{U}(\mathrm{n}, \mathrm{f})^{140} \mathrm{La}$ | $4.5 \times 10^{-12}$ | 1.4 | 6.7 |
| ${ }^{63} \mathrm{Cu}(\mathrm{n}, \alpha){ }^{60} \mathrm{Co}$ | $6.7 \times 10^{-15}$ | 6.0 | $1.6 \times 10^{1}$ |
| ${ }^{56} \mathrm{Fe}(\mathrm{n}, \mathrm{p}){ }^{56} 6_{\mathrm{Mn}}$ | $1.8 \times 10^{-14}$ | 5.5 | $1.6 \times 10^{1}$ |
| $24 \mathrm{Mg}(\mathrm{n}, \mathrm{p})^{24} \mathrm{Na}$ | $3.0 \times 10^{-14}$ | 6.7 | $1.7 \times 10^{1}$ |
| $48^{T i}(\mathrm{n}, \mathrm{P})^{48} \mathrm{Sc}$ | $7.6 \times 10^{-15}$ | 6.7 | $1.7 \times 10^{1}$ |



## C. Solution Spectrum for V-1 Facility

Foil reaction data used as the input for $V-1$ solution are presented in Table 5. The first solution spectrum, obtained after 16 iterations, had a standard deviation of $\sigma= \pm 27 \%$. At the 16 th iteration the ${ }^{48} \mathrm{Ti}(\mathrm{n}, \mathrm{p})^{48} \mathrm{Sc}$ had a percent deviation of +36.53 , the highest of all reactions used. Following the rejection of ${ }^{48} \mathrm{Ti}$ activity, the code reached the final solution in 9 iterations (see Appendix C).

The V-1 initial solution spectrum (prior to the rejection of ${ }^{48} \mathrm{Pi}_{\mathrm{Ti}}(\mathrm{n}, \mathrm{p})^{48} \mathrm{Sc}$ ) and final spectrum (after the rejection of the ${ }^{48} \mathrm{Ti}_{\mathrm{Ti}(\mathrm{n}, \mathrm{p})}{ }^{48}$ Sc reaction) are presented in Figures 5 and 6, respectively. The SAND-II integral flux values indicated that the flux of fast neutrons with $\mathbb{E} \geqslant 0.11 \mathrm{MeV}$ was $6.74 \times 10^{14}$ $\mathrm{n} / \mathrm{cm}^{2} \mathrm{sec}$. This is 4.3 times greater than the fast flux (with $E \geqslant 0.11 \mathrm{MeV}$ ) for the R-3 facility. However, the integral flux of neutrons with $\mathrm{E} \geqslant 1 \mathrm{MeV}$ was $1.18 \times 10^{13} \mathrm{n} / \mathrm{cm}^{2} \mathrm{sec}$. This value, comparable to the integral flux of the R-3 facility, is in fair agreement with the $A L R R$ measured value of $2 \times 10^{13} \pm 25 \% \mathrm{n} / \mathrm{cm}^{2}$ $\sec ^{*}$ 。

## D. Error Analysis

Errors affecting the final solution spectrum can be divided into two groups. The first group consists of errors in the

[^6]Table 5. $V-1$ saturated measured activities and reactions response boundaries

| Reaction | $\begin{gathered} \text { measured } \\ \text { activity } \\ \text { (dps/nucleus) } \end{gathered}$ | reaction response boundaries (MeV) |  |
| :---: | :---: | :---: | :---: |
|  |  | lower | upper |
| ${ }^{27} \mathrm{Al}(\mathrm{n}, \alpha)^{24} \mathrm{Na}$ | $2.1 \times 10^{-14}$ | 6.0 | $1.6 \times 10^{1}$ |
| $197^{\text {Au }(n, \gamma)}{ }^{198} \mathrm{Au}$ | $5.7 \times 10^{-8}$ | $4.2 \times 10^{-6}$ | $6.3 \times 10^{-6}$ |
| ${ }^{59} \mathrm{Co}(\mathrm{n}, \gamma){ }^{60} \mathrm{Co}$ | $2.7 \times 10^{-9}$ | $1.9 \times 10^{-6}$ | $1.7 \times 10^{-4}$ |
| ${ }^{63} \mathrm{Cu}\left(\mathrm{n}, 8\right.$ ) ${ }^{64} \mathrm{Cu}$ | $1.4 \times 10^{-10}$ | $8.4 \times 10^{-7}$ | $2.6 \times 10^{-1}$ |
| $58_{\mathrm{Fe}}(n, \gamma){ }^{59} \mathrm{Fe}$ | $5.2 \times 10^{-11}$ | $1.3 \times 10^{-6}$ | $6.6 \times 10^{-2}$ |
| ${ }^{54} \mathrm{Fe}(\mathrm{n}, \mathrm{p})^{54} \mathrm{nn}$ | $1.1 \times 10^{-12}$ | 2.3 | 8.2 |
|  | $3.2 \times 10^{-12}$ | 5.5: $\times 10^{-12}$ | 6.0 |
| $58^{\mathrm{Ni}(\mathrm{n}, \mathrm{p})}{ }^{58} \mathrm{Co}$ | $1.6 \times 10^{-12}$ | 2.0 | 8.2 |
| $9^{7} \mathrm{Zr}(\mathrm{n}, 2 \mathrm{n})^{89} \mathrm{Zr}$ | $3.7 \times 10^{-14}$ | $1.3 \times 10^{1}$ | $1.9 \times 10^{1}$ |
| ${ }^{46} \mathrm{Ti}(\mathrm{n}, \mathrm{p})^{46} \mathrm{Sc}$ | $1.9 \times 10^{-13}$ | 4.1 | $1.3 \times 10^{1}$ |
| $47_{\text {Ti }}(\mathrm{n}, \mathrm{D})^{47} \mathrm{Sc}$ | $3.4 \times 10^{-13}$ | 1:8 | 8.2 |
| $232_{\operatorname{Th}(\mathrm{n}, 1)^{140} \mathrm{~L} 2}$ | $1.2 \times 10^{-12}$ | 1.4 | 8.2 |
| $\left.{ }^{238}{ }_{U(n, f}\right)^{140}{ }_{\text {La }}$ | $5.0 \times 10^{-12}$ | 1.2 | 7.4 |
| ${ }^{63} \mathrm{Cu}(\mathrm{n}, \alpha){ }^{60} \mathrm{Co}$ | $9.2 \times 10^{-15}$ | 6.0 | $1.5 \times 10^{1}$ |
| $55_{\mathrm{Fe}(\mathrm{n}, \mathrm{p})}{ }^{56} \mathrm{Mm}$ | $2.8 \times 10^{-14}$ | 5.5 | $1.5 \times 10^{1}$ |
| $24_{15}(n, p){ }^{24} 4^{2}$ | $4.1 \times 10^{=14}$ | 6.0 | $1.6 \times 10^{i}$ |
| ${ }^{43} \mathrm{Pi}(\mathrm{n}, \mathrm{p})^{48} \mathrm{Sc}$ | $1.3 \times 10^{-14}$ | 6.0 | $1.7 \times 10^{1}$ |



input data, and the second consists of errors due to the fact that the mathematical nodel cannot yield a unioue solution. To quantify the errors in the final solution swectra obtsined for R-3 and V-1 facilities, a SAND-II Fonte Cerlo error analysis code was run. The code combines uncertinties in the cross sections and input activities to provicie estimates of uncertzinty for the solution spectrum ( 43,47 ). For a given reaction the exact volue of the cross section uncerteinity as a function of energy, $\pm \Delta \sigma_{i}(E)$, is not known. To provice an estimate for the uncertainties in the cross sections, the energy renge $\mathcal{B}$ is divided into 15 intervals and within each of these intervals, an average error is assumed to apply uniformly to each value of the cross section within the energy range (43). The Konte Carlo code selects values of input activities and cross sections with assigned errors (see Table 6) for $\approx$ preselected number of SAND-II runs. The values of input activities and cross sections, corresponding to the various runs, are used to generate sets of solution spectra, which, by virtue of their differences, provide the error estimates.
 tion spectra ore presented in Figures 7 and 8 (se $\begin{gathered}\text { also Appen- }\end{gathered}$ dix D). In both cases, the error band is wicest at enengy intervels not well covered by the reactions. This is zenticulerIy noticeable at eneraies below the cadmium cut-one ant the rev region. Within the enersy renge of 1 to 10 Kev , themenituce

Table 6. SAND-II evaluated cross section error assignment


* $1^{-10 / 4^{-7}}=$ from $1 \times 10^{-10}$ to $4 \times 10^{-7} \mathrm{MeV}$

Table 6. (continued)



Figure 7. R-3 solution spectrum error analysis


Figure 7-A: R-3 input and solution spectra



Figure 8-A. $V-1$ input and solution spectra
of the final solution is generally within $\pm 10 \%$. But at energies greater than 10 MeV , the error band widens. This is due to greater uncertainties associated with the velues of $\sigma_{i}(E)$ at higher energies. Another contributing factor is the lesser coverage of these high energy regions by the reactions available to be used.

The results of several SAND-II runs involving different sets of experimentally measured activities showed that generalIy aiter a number of iterations the ${ }^{48} \mathrm{Ti}(n, p)^{48}$ Sc reaction was rejected. The percent deviation of the ${ }^{48} \mathrm{Sc}$ activity was positive for all runs and had a velue ranging from $9 \%$ to $38 \%$. Since the input spectra and the cross section values were not changed from run to run, it was concluded that the values of the ${ }^{48} \mathrm{Ti}(n, p){ }^{48}$ Sc reaction cross section with uncertainties up to $\pm 50 \%$ (see Table 6) were the main cause of the rejection. The rejection of the ${ }^{48}$ Ti $(n, p)^{48}$ Sc reaction data and such other reaction data a.s $9^{90} \mathrm{Zr}(\mathrm{n}, 2 \mathrm{n})^{89} \mathrm{Zr}$ and ${ }^{63} \mathrm{Cu}(\mathrm{n}, \alpha)^{60}$ Co has been reported by other investigators*. In these cases a higher level of uncertainty in the value of cross sections was found to be the main cause of the rejection. In fact, as may be noted from Table 6, the uncertainty of the above reactions is

[^7]EEnerally higiner than the corresponding uncertainty for the other reactions used.

The values of the R-3 facility differential neutron flux, although reasonable up to $\mathrm{E} \simeq 10 \mathrm{MeV}$, seemed to be rather high for energies greater then 10 MeV . To study the spectral shape of the R-3 flux at higher energies, the SAND-II solution was compored with a fission spectrum $\varphi_{R-3}^{*}(\mathbb{Z})$, obtained from the pollowing Equation:

$$
\Phi_{R-3}^{*}(\mathbb{E})=\Phi_{R-3} \times 0.770 E^{1 / 2} \exp (-0.776 E)
$$

where $\Phi_{R-3}=1.547 \times 10^{14} \mathrm{n} / \mathrm{cm}^{2} \mathrm{sec}$ is the integral flux of the neutrons with $E \geqslant 0.11 \mathrm{MeV}$ and E is the neutron energy in MeV . As shom in Figure 9, at energies greater than approyimateIy 13 MeV , the actual $\mathrm{R}-3$ flux, $\varphi_{\mathrm{R}-3}(\mathrm{E})$, becomes increasingly greater than the fission flux $\Phi^{*} R-j^{(B)}$. As indicated in the preceding pages, part of this error is due to the greater cross section uncertainties and to the lack of sufficient foil covera.ge at higher enengies, Another contributing factor to the Giscrepancy is the shape of the input spectrum used for the R-3 ミacility (Figure 10) 。

Sterting from the input spectrum, the SAIFD-II code reached E. reesonaible final solution in a small number of iterations. To reach the solution, however, the input spectrum was extensiveIy reduced at the low enezoy poation of the spectrun, as is

Figure 9. High energy portion of the $\mathrm{R}-3$ solution spectrum compared with a fission spectrum


Figure 10. High energy portion of the R-3 input spectrum

shown in Figure 7-A. This resulted in what may be termed a "spectrum distortion": a reduction in magnitude at the low energy portion and an increase in magnitude for $E>10 \mathrm{MeV}$.

A similar trend, although to a lesser extent, was observed for the $\mathrm{V}-1$ solution spectrum. At energies greater than 10 MeV , the actual spectrum approached the values of $\phi^{*} \mathrm{~V}_{-1}(\mathbb{E})$ defined by

$$
\varphi_{V-1}^{*}(E)=\Phi_{V-1} \times 0.770 E^{1 / 2} \exp (-0.776 E)
$$

with $\Phi_{\mathrm{V}_{-1}}=6.745 \times 10^{14} \mathrm{n} / \mathrm{cm}^{2} \mathrm{sec}$ being the integral flux of the $\mathrm{V}-1$ neutrons with $\mathrm{E} \geqslant 0.11 \mathrm{Mev}$. The values of $\varphi_{\mathrm{V}-1}(\mathbb{E})$ and
$\Phi^{*}{ }_{V-1}(\mathrm{E})$ are plotted in Figure 11 . The high energy portion of the $\mathrm{V}-1$ input spectrum is illustrated in Figure 12.

Figure 11. High energy portion of the $V-1$ solution spectrum compared with a fission spectrum


Figure 12. High energy portion of the $\overline{\mathrm{V}}-1$ input spectrum


## VII. SUMMARY AND CONCLUSIONS

The state of the art of multiple foil unfolding techniques has been reviewed. Of the procedures which have been explored and are available for consideration, Ringle (26) found that the Cross Section Expansion method provided the best agreement between actual (test) spectra and deduced profiles in the 2-30 MeV region selected for analysis. Of the three computer codes recommended for use, Dierckx $(36,37)$ identified the SAND-II code as being somewhat superior to SPECTRA and CRYSTALL BALL. Consequently, the SAND-II code was selected for use in exploring the neutron energy distribution in the two ALRR irradiation facilities $\mathrm{R}-3$ and $\mathrm{V}-1$.

Activation detectors providing seventeen reactions were employed covering an energy range from 0.5 eV to 20 MeV . Because of the interest in exploring the higher energy region, twelve of the reactions had threshold energies in the MeV region.

The unfolding code iterates the flux distribution from a selected input spectrum until the calculated foil activations agree, within a predetermined limit of $5 \%$ total deviation from the experimentally determined values. The final solution is heavily dependent on the input spectrum due to the fact that the seventeen pieces of input activation data are much less than the 100 energy group flux amplitudes which must be specified to establish the spectrum. The number of iterations required to
complete the analysis is also dependent on the input spectra. For example, if by chance the selected input spectrum is correct the calculated and input activations will agree and no modification of the amplitude in any energy group will be required.

The selection of an appropriate input spectrum is facilitated by the existence within the code of a collection of potentially useful spectra. A choice of input spectrum was made for each of the two cases by testing all possibilities and selecting the one which required the smallest number of iterations to achieve the given level of agreement between measured and calculated activations. The input spectra for the R-3 and V-1 cases are shown in Figures 1 and 2 respectively. It will be noted that the input spectrum chosen by this method for the $\mathrm{V}-1$ is somewhat "harder" than is the case for R-3.

The code measures the inconsistency between the evolving best fit spectrum and the individual contributions from the detectors. When a contribution deviates from the best fit value in excess of a predetermined amount, the contribution is dropped and the fitting procedure is carried out using the remaining data. Figures 3 and 4 show the spectrum obtained with and without a contribution from the ${ }^{48} \mathrm{Ti}(n, p){ }^{48} \mathrm{Sc}$ reaction. The curves are seen to be identical which is not surprising since a number of other detectors are sensitive over part or
all of the same energy range. The consistency which results when the ${ }^{48} \mathrm{Ti}(n, p){ }^{48} \mathrm{Sc}$ contribution is dropped, is improved to the extent that the deviation is reduced from 3.87 to $3.36 \%$ (Appendix B). It will be observed that the final solution for the R-3 spectrum (Figure 4) shows a significant decrease in the number of neutrons in the energy range between 0.5 MeV and 20 MeV as compared to the input spectrum (Figure 1). Figure 7a provides a direct comparison between the input and unfolded spectra for the R-3 facility.

As might be expected, the ${ }^{48} \mathrm{Ti}(n, p){ }^{48}$ Sc contribution was also rejected during the analysis of the data obtained in the V-1 facility. As before, the spectra obtained with and without the ${ }^{48}$ Ti reaction (Figures 5 and 6) are identical. In this case the standard deviation after nine iterations is reduced from $11.3 \%$ to $4.9 \%$ (Appendix C). Although there is some evidence of structure in the spectrum for higher energies, the most striking observation is that the initially high neutron density for energies greater than 0.2 MeV (Figure 2) is adjusted downward in the energy range between 0.2 MeV and 20 MeV (Figures 6 and 8a).

A comparison of the spectra for the two facilities (R-3 and $V-1$ ) as shown in Figures 4, 6, 7, 7a, 8, and 8a reveals that much of the difference between the two distributions can be explained by the difference in the two input spectra. The higher relative neutron density in the energy range between
$10^{-3} \mathrm{MeV}$ and 0.5 MeV for $\mathrm{V}-1$ as compared to $\mathrm{R}-3$, as shown in Figures 4 and 6 or Figures 7 and 8, corresponds to the higher neutron density characteristics of the input spectrum for $\mathrm{V}-1$ as shown in Figure 2. It will be noted that in the energy range between $10^{-3}$ and 0.5 MeV the modifications of the input spectra for the two cases must come from contributions from $(n, x)$ reactions with the ${ }^{58} \mathrm{Fe},{ }^{59} \mathrm{Co},{ }^{63} \mathrm{Cu}$ and ${ }^{197} \mathrm{Au}$. Since all the detector foils were shielded by cadmium, they were responsive only to neutron energies greater then about $5 \times 10^{-7} \mathrm{MeV}$. In the energy range of interest, $1 \times 10^{-3}$ to $5 \times 10^{-1} \mathrm{MeV}$, the gold cross section decreases monotonically from 5 barns to 0.2 barns with the bulk of its activation originating from the $3 \times 10^{4}$ barn resonance at $4.8 \times 10^{-6} \mathrm{MeV}$. Thus it is not to be expected that the integral gold activation data can provide a significant correction in this relatively narrow energy region of interest. The situation for cobalt is similar in that resonance contributions are significant ( 7000 barns and 260 barns) at $1.35 \times 10^{-4} \mathrm{MeV}$ and between $4.3 \times 10^{-3}$ and $5 \times 10^{-3} \mathrm{MeV}$. In this case roughly two energy groups around $4.5 \times 10^{-3} \mathrm{MeV}$ would be affected and examination of Figure 6 seems to support this conclusion. Although iron and copper have resonances they are not as pronounced as those just described. They do however fall within the region of interest; the iron resonances are found between $8 \times 10^{-3}$ and $5 \times 10^{-1} \mathrm{MeV}$ while those for copper are found between $2 \times 10^{-4}$
and $5 \times 10^{-2} \mathrm{MeV}$. For both of these detectors the cross section is roughly constant with energy which implies that their effect will be roughly proportional to the width of the energy range of interest or $0.5 / 20=0.025$. Thus the corrections would once again be small. The overall conclusion is that with the detectors used no significant modification of the input spectrum is to be expected in the energy region between $1 \times 10^{-3}$ and $5 \times 10^{-1} \mathrm{MeV}$.

The difference between the spectra in the two facilities was explored with the aid of the differential flux curves (Figures $1-8 a$ ) and the values of both the normalized and the absolute integral fluxes tabulated in Appendices $B$ and $C$ corresponding to $\mathrm{R}-3$ and $\mathrm{V}-1$ respectively. The tabulated data gives the integral fluxes bbove selected or specified energy values. The absolute values provide information reflecting spacial and configuration dependent factors. For exampe since $\mathrm{V}-1$ is in the center of the core and $\mathrm{R}-3$ is in the moderator/reflector outside the core it would be expected that the magnitude of the flux in $V-1$ would exceed that found in $R=3$. In addition it would be expected that the energy dependent flux (the spectrum) would be "harder" in $V-1$ than in $R-3$ due to the flux-trap nature of the coaxial fuel element in the $V-1$ position and the proximity of moderator to the $\mathrm{R}-3$ position. On the other hand the fact that the moderator and coolant are heavy water would tend to smooth out rapid or local variations in the
spectrum. The integral normalized fluxes should be better indicators of the energy dependence of the fluxes on the two facilities. For convenience selected data has been abstracted from the Appendices and is presented in table 7.

The interesting figures are the ratios of the integral fluxes in $\mathrm{V}-1$ to those in $\mathrm{R}-3$ for the various indicated lower energies. The ratio of absolute integral fluxes is 0.90 for neutrons with energies greater than 2 MeV and is 4.36 for neutrons with energies greater than 0.11 MeV (the energy range of greatest importance when considering neutron damage). From an experimental point of view the $\mathrm{V}-1$ facility is clearly the better place in which to carry out fast neutron effect studies. This conclusion is not as easily drawn from examination of the differential flux curves (Figures 4 and 6) although $V-1 / R-3$ differential flux ratios do provide supporting information.

Dxamination of the relative integral flux ratios reveals that for neutron energies above 2 MeV , above 1 MeV and above 0.5 MeV , the relative neutron density is greater in $\mathrm{R}-3$ than in $\mathrm{V}-1$. Only for energy ranges with lower limits below 0.11 MeV does $V-1$ offer a harder spectrum. The total flux in $V-1$ is seen to be about 2.5 times Iarger than the corresponding fiux in $\mathrm{k}-\overline{3}$.

To establish confidence in the results of the SAND-II analysis, a Monte Carlo error study was carried out using the uncertainties in detector cross sections and experimentally determined activations as input. The results of the analyses

Table 7. Integral flux data taken from Appendices B and C

| Energy MeV | 2 | $0.5 \quad 0.11$ | $1.275 \times 10^{-3} 1.350 \times 10^{-5}$ | $1.275 \times 10^{-6}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{R}-3\left\{\begin{array}{l} \text { Absolute } \\ \text { Relative } \end{array}\right.$ | $\begin{aligned} & 5.869 \times 10^{12} \\ & 1.049 \times 10^{-2} \end{aligned}$ | $\begin{aligned} & 3.860 \times 10^{13} 1.547 \times 10^{14} \\ & 6.896 \times 10^{-2} 2.764 \times 10^{-1} \end{aligned}$ | $\begin{aligned} & 2.793 \times 10^{14} 3.990 \times 10^{14} \\ & 4.989 \times 10^{-1} 7.128 \times 10^{-1} \end{aligned}$ | $\begin{aligned} & 4.574 \times 10^{14} \\ & 8.172 \times 10^{-1} \end{aligned}$ |
| $\mathrm{V}-1\left\{\begin{array}{l} \text { Absolute } \\ \text { Relative } \end{array}\right.$ | $\begin{aligned} & 5.308 \times 10^{12} \\ & 3.753 \times 10^{-3} \end{aligned}$ | $\begin{aligned} & 7.262 \times 10^{13} 6.745 \times 10^{14} \\ & 5.135 \times 10^{-2} 4.769 \times 10^{-1} \end{aligned}$ | $\begin{array}{ll} 1.138 \times 10^{15} & 1.263 \times 10^{15} \\ 8.048 \times 10^{-1} & 8.927 \times 10^{-1} \end{array}$ | $\begin{aligned} & 1.348 \times 10^{15} \\ & 9.531 \times 10^{-1} \end{aligned}$ |
| $\frac{(V-1)}{(R-3)} \text { Abs. }$ | 0.904 | 1.881 4.360 | $4.074 \quad 3.165$ | 2.947 |


| $\frac{(V-1)}{(R-3)}$ Rel | 0.358 | 0.745 | 1.725 | 1.613 | 1.252 | 1.166 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

are shown in Figures 7 and 8. Comparison between Figures 7 and $7 a$, and 8 and $8 a$ shows the uncertainty zones of the unfolded spectra.

The computed values of the high, low, mean and true flux values and the percent variations from the mean for the fluxes in $\mathrm{R}-3$ and $\mathrm{V}-1$ are presented in Appendix $D$. It will be noted that the percent uncertainties are higher by a factor of 2 for $\mathrm{V}-1$ fluxes as compared to $\mathrm{R}-3$ fluxes in the energy range below $1 \times 10^{-6} \mathrm{MeV}$. For energies between $1 \times 10^{-6}$ and $3 \times 10^{-1}$ MeV and $\mathrm{V}-1$ flux uncertainties are higher by approximately $25 \%$ and for energies greater than about $8 \times 10^{-1} \mathrm{MeV}$ (Table 8). In the main, these peculiarities correlate well with the differences between the mean and true flux values. For energies below about $1 \times 10^{-6} \mathrm{MeV}$ the mean value of the flux in $\mathrm{R}-3$ is greater than the true value while for the flux in $V-1$ the reverse is true. For energies in the interval between $1 \times 10^{-6}$ and $3 \times 10^{101} \mathrm{MeV}$, the mean values of the flux are smaller than the true values of the flux for both facilities. For energies between 1 and 6 MeV there is some oscillation but in the main whatever is the case for one facility the reverse is true for the other: Finally, for energies above 6 MeV the mean flux is greater than the true flux for both facilities.

There is a significant departure in the patterns for energies in the range $1 \times 10^{-1}$ to 1 MeV and for energies greater than 10 MeV . For these energy regions, the variations are significantly larger than they are for adjacent energies,

Table 8. Differential flux data taken from Appendices $B$ and $C$

| Energy MeV | 2 | 0.5 | 0.11 | $1.275 \times 10^{-3}$ | $1.350 \times 10^{-5}$ | $1.275 \times 10^{-6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{V}-1\left\{\begin{array}{l}\text { Absolute } \\ \text { Relative }\end{array}\right.$ | $\begin{aligned} & 3.399 \times 10^{12} \\ & 2.403 \times 10^{-3} \end{aligned}$ | $\begin{aligned} & 2.597 \times 10^{14} \\ & 1.837 \times 10^{-1} \end{aligned}$ | $\begin{aligned} & 3.044 \times 10^{15} \\ & 2.152 \times 10^{0} \end{aligned}$ | $\begin{aligned} & 1.358 \times 10^{16} \\ & 9.599 \times 10^{0} \end{aligned}$ | $\begin{aligned} & 1.054 \times 10^{18} \\ & 7.456 \times 10^{2} \end{aligned}$ | $\begin{aligned} & 1.200 \times 10^{19} \\ & 8.485 \times 10^{3} \end{aligned}$ |
| $\mathrm{R}-3\left\{\begin{array}{l} \text { Absolute } \\ \text { Relative } \end{array}\right.$ | $\begin{aligned} & 3.307 \times 10^{12} \\ & 5.908 \times 10^{-3} \end{aligned}$ | $\begin{aligned} & 1.685 \times 10^{14} \\ & 3.011 \times 10^{-1} \end{aligned}$ | $\begin{aligned} & 3.517 \times 10^{14} \\ & 6.283 \times 10^{-1} \end{aligned}$ | $\begin{aligned} & 1.691 \times 10^{16} \\ & 3.021 \times 10^{1} \end{aligned}$ | $\begin{aligned} & 1.624 \times 10^{18} \\ & 2.901 \times 10^{3} \end{aligned}$ | $\begin{aligned} & 1.581 \times 10^{19} \\ & 2.825 \times 10^{4} \end{aligned}$ |
| $\frac{(V-1)}{(R-3)} A b s$ | 1.028 | 1.541 | 8.655 | 0.803 | 0.649 | 0.759 |
| $\frac{(v-1)}{(R-3)} \mathrm{Rel}$. | 0.407 | 0.610 | 3.425 | 0.318 | 0.257 | 0.300 |

As noted before, the detectors with energy sensitivities in the ranges $1 \times 10^{-2}$ and $5 \times 10^{-1} \mathrm{MeV}$ were sensitive over a broad energy range and no detectors were used with energy sensitivities greater than 20 MeV , or less than $5 \times 10^{-7} \mathrm{MeV}$; consequently it is not too surprising that the SAND-II iteration procedure was unable to accomplish necessary corrections and reduce the variations in these regions.

The four wide range ( $n, \gamma$ ) detectors are the only detectors used with sensitivities between energies of $4 \times 10^{-4}$ and $5 \times 10^{-1} \mathrm{MeV}$. This fact. provides an explanation for the remaining principal features of the Monte Carlo error analysis viz. the large variations between $1 \times 10^{-1}$ and 1 MeV . For those energies where suitable detectors were not available or, if available, were not used, any agreement between a smooth curve based on an iterated input spectrum and the true spectmu would be accidental Moreover; the variation between highest and lowest values could be large.

As would be expected the change of the flux variation with energy reflects the changing uncertainty in the cross sections, as reported in Table 6.

The final unfolded spectra for $\mathrm{R}-3$ and $\mathrm{V}-1$ were compared with fission spectra as shown in Figures 9 and 11. It was observed that contrary to reason and expectations the unfolded spectra for energies greater than about 10 MeV had high energy components greater than would an appropriately normalized
fission spectrum. Although these observations can be explained in terms of cross section uncertainties or lack of sensitivity of available detectors in this high energy region another possibility is that the original input spectra are involved. Inspection of Figure 7a shows how the low energy portion of the input spectrum was drastically reduced which implies that for a constant neutron population, the high energy portion is relatively augmented. Had suitable detectors been available the iteration procedure would have provided appropriate corrections. The significance for this work is that the indicated high energy neutron flux is in error. However, the importance of the effect is academic. From the point of view of radiation damage effects, the magnitude of the integrated flux for energies above 0.11 MeV and the $(\mathrm{V}-1) /(\mathrm{R}-3)$ ratios are essentially unchanged from the tabulated values if contributions for ener gies above 10 MeV are ignored.

The results clearly indicates: (a) the importance of the input spectrum in determining the shape of the final spectrum in energy regions not adequately explored by detection reactions; (b) the value of having a number of detectors sensitive in the same energy range as revealed by the stability of the spectrum to rejection of the ${ }^{48}$ Ti contributions; (c) the relative ineffectiveness of a detector, like ${ }^{63} \mathrm{Cu}$, sensitive over a wide energy range as demonstrated by the lack of change in the spectrum for the $V-1$ facility in the energy range $4 \times 10^{-4}$ to
$2 \times 10^{-2} \mathrm{MeV}$; (d) the ability of $S A N D-I I$ to rapidly modify an incorrect input spectrum in accordance rith information obtained from a number of detectors as shown by the results from both sets $O I$ data in the energy range from 0.5 to 20 MeV .
VIII. SUGGESIIONS FOR FUTURE WORK

The input spectra in this work were "best guess-empirical" distributions. Their selection, among the distributions tested, was based primarily on the fact that they yielded the minimum number of iterations required to reach a final solution. Hence, future use of the unfolding codes could be improved by using more suitable input spectra established by sophisticated calculations.

A related area of interest is a study of the sensitivity of the unfolding code results to the choice of input spectrum. One possible approach would be as follows:

1 - select a number of different spectra $\varphi(\mathbb{E})$, actual or purely mathematical, to serve as the unknown spectrum to be determined by the unfolding procedure;

2 - calculate the reaction rates of several foils, using these spectra;

3 - choose a number of input spectra $\varphi_{1}^{0}(\mathbb{E})$ for each $\varphi(\mathbb{E})$;
4 - quantify the variation of the solution spectra $\tilde{\varphi}(\mathbb{F})$ as a function of the variation in $\varphi_{I}^{O}(\mathbb{I})$. This may be accomplished by studving the following expressions:

$$
\begin{aligned}
\Delta \varphi_{1}(\mathbb{E})= & \tilde{\varphi}(\mathbb{E})-\phi_{1}^{0}(E)=\sum_{i, j} k_{i j}\left[A_{s p i}-\right. \\
& \left.-\int_{0}^{\infty} \sigma_{i}^{\infty}(\mathbb{E}) \varphi_{1}^{0}(\mathbb{E}) d \mathbb{E}\right]
\end{aligned}
$$

$$
\begin{aligned}
& \left.\Delta \varphi_{2}=\tilde{\varphi}(B)-\phi_{2}^{0}(B)=\sum_{i, j} k_{i j}\left[A_{S p i}-\int_{0}^{\infty} \sigma_{i}(B) \varphi_{2}^{0}(B) d\right]\right] \\
& \vdots \\
& \Delta \varphi_{I}=\tilde{\varphi}(D)-\varphi_{I}^{0}(B)=\sum_{i, j} k_{i j}\left[A_{\sin }-\int_{0}^{\infty} \sigma_{i}(B) \varphi_{I}^{0}(\mathbb{B}) d Z\right]
\end{aligned}
$$

which lead to

$$
\left(\delta_{1}^{2}\right)=\left(\Delta \phi_{1}\right)^{2} / \tilde{\varphi}(\square)
$$

 $j=1, \ldots$, Inciactes the energy interval, and $A_{\text {spit }}$ is the reaction rate calculated with $\varphi(\overrightarrow{\text { a }}$ ) for reaction $i$.

This sensitivity study could be complemented by further investigating the "importance" of the response functions in energy regions which are not sufficiently covered by the detertore.

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XI. APPEITDIX A


Figure 13. R-3 facility


Pisure li4. V-i Iacility
XII. APPEITDIX B




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## SULTTIMT SPECTRTM OGTJTNEU AFTER 2 TTERATIORS

| ABSTICUTE VALUES |  |  | NURMALTLED TU T．OUE＝10－MET |  |  |
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| 1．000E－10 | 2.032 El 19 | 5．596E 14 | $3.631 E 04$ | 1．000E 00 | $1.477 \mathrm{E}-01$ |
| －T．000 - －${ }^{\text {a }}$ | 1．951E 20 | $5.506 \mathrm{E}^{14}$ | 3．504E－05 | I．OC0E DO | $1.477 E=01$ |
| 1．OJOE－O8 | 4.393 E 20 | 5.579 E 14 | 7．85je 05 | 9．968E－01 | 1．482E－01 |
| 2．3005－09 | 4．0344E 20 | $5.322 E 14$ | 8.298 O | 9．860E－01 | 1．497E－01 |
| 5：00．15－1，3 | $3.559 E 20$ | $5.396 E 14$ | $6.271 E T 5$ | 9．479F－01 | 1．532E－01 |
| 7．60Ve－03 | 2.710 E 20 | 5． T ¢JE 14 | 4．878E 05 | $9.479 E-01$ $9.289 E-01$ | 1．590E－01 |
| 1． 5 5－9E－07 | 1.949 E 20 | 5.198 CL 14 | 3．483E JS |  | $1.624 E=01$ |
| 1．7心リヒーご | $1.230 E 20$ | S．C71E T4 | $2.287 E ~ O S$ $1.497 E \sim 5$ | B． $703 \mathrm{E}-01$ | $1.659 \mathrm{E}-01$ |
| $2.53 n 6-07$ | $8.376 E 19$ $5.599 E 19$ | $4.932 E 14$ $4.878 E 14$ | $1.497 E ~$ $9.985 E$ 0.9 | 8． $710 \mathrm{E}-01$ | $1.695 \mathrm{E}-01$ |
| 3．30UF－07 | 5．598E 19 |  | $6.597 E$ O4 | B．3405 $=01$ | L．TくロE－01 |
| 5．5NTECT | $3.070 E^{19}$ $2.372 E 15$ | 4．783E 4.6 | 4.239 O | B． $356 \mathrm{E}-01$ | $1.768 \mathrm{E}-01$ |
| 3．4U．JE－0i L． $275=-16$ | 2.3 .2519 $1.531 E 19$ | $4.573 \mathrm{E} \quad 14$ | 2．825E 34 | 8．171c－01 | 1．808E－91 |
| 1．J07－－45 | 1.0 ¢E 19 | $4.474 \mathrm{E}^{-14}$ | T．837E 04 | T．795E＝0t | 1．848E＝01 |
| 2．300E－06 | 7．0U3E 18 | 4.382 E 14 | 1．251E 04 | 7．829E－01 | 1．887E－01 |
| 4．2－05－46 | $4.760 E 18$ | $4.230 E 14$ | $8.506 E 03$ | 7．6485－01 | 1．932E－01 |
| 6．350 $=06$ | 3．2J7E I | 4．TJ3E I 4 | $5.890 \mathrm{E}^{03}$ | 7．474E＝01 | 1．977E－U1 |
| 9．2005－06 | 2．279E 18 | 4.097 E i4 | 4.072 E 33 | 7. | 2．023E－01 |
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| 3．0こ0＝－05 | 7．256E 17 | $3.770 E: 4$ | $1.298 E ~$ <br> 8.205 | $6.737 E-01$ $6.542 E-C 1$ | 2．258E－01 |
| 4．5．1．15－05 | 4.572 E 17 | $3.661 E 14$ | 8．205E J2 | $6.542 E-C 1$ $6.345 E-01$ | 2．328E＝01 |
| 6．700E－05 | 3.078 ETT | $3.551 E 14$ | $5.479 E$ J2 | 6．17JE－01 | 2．302E－01 |
| 1．GOOE－S4 | 2．170E 17 | 3．450E 14 | 3．712E 32 | $6.173 E-01$ $6.038 E-01$ | $2.447 E-01$ |
| 1.3 O $1.0-C 4$ | 1.729 E 17 | 3.379 E 14 | 3.090 E .22 | 6．939E－01 | 2．441E－OI |
| 1．70ヶご04 | $1.375 \mathrm{ET7}$ | 3.310614 | 2．438E VR | $5.729 E-01$ $5.807 E-01$ | 2．544E－01 |
| 2．2いいこ－04 | 1.114 E 17 | $3.250 E 14$ | $1.991 E 02$ | 5．5日7E－01 | $2.577 \mathrm{E}-01$ |
| 2．90， | 3.773 E is | $3.193 E 14$ | 1．508E 22 | 5．562E＝O1 |  |
| 3．5\％ | ह．5 515 | $3.173 E I 4$ | 1．193E JZ | 5．562E－OI | 2．8．708E－01 |
| 4．5；） $5-04$ | $5.153 E 16$ | $3.053 \mathrm{E} \quad 14$ | $9.209 E 01$ | $5.454 E-01$ $5.339 E-01$ | $2.708 E-01$ $2.766 E-01$ |
| 5．7：0¢－C4 | $3.771 E 16$ | 2.992 E l 4 | $6.737 E 01$ | 5．339E－01 | 2．706E－01 |
| 7．50 $5-5$ | 2.870 E 16 | 2．0T8ET4 | 5.137601 | $5 \cdot 215 \mathrm{E}-01$ | 2．832E－01 |
| 7．01）E－04 | 2．196E 16 | 2.861514 | 3.924 EL | 5－112E－01 | $2.899 E-01$ $2.960 E-01$ |
| 1．2イラミー03 | $1.6 ว 1516$ | 2．7．72E 14 | 3．022E OL | $4.988 \mathrm{E}-01$ | 2．960E－01 |
| $1: 5 寸 \Psi=-03$ | 1.351515 | $2.737 E-4$ | 2．415E 1.856 |  | 3．080E－01 |
| 2．：）${ }^{\text {Pe－03 }}$ | 1.039 c i6 | $2.683 E 14$ | $1.856 E ~$ <br> $1: 4275$ <br> 1 | $4.794 E-01$ $4.684 E-01$ | 3．155E－01 |
|  | 7．985E 5 | 2.610 LC | 1：427501 | 4．664E－01 | 3．155－01 |

ABSDLUTE VALUFS NCIRMALILEIT TO 1. OOE-10 NEV



INTEKIA RESHIITS AFTER O ITERATIDNS

|  |  | $\begin{array}{r} \text { NDM } \\ 5.00 \mathrm{P} \\ \text { ACTI } \end{array}$ | $\begin{aligned} & \text { INTL } \\ & \text { ERCENT } \\ & \text { IIIY } \end{aligned}$ | RATIO | $\begin{aligned} & \text { DEVIATIUN } \\ & \text { OF MEASURED } \\ & \text { FROM } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | LIMITS | (MEVT | MEASURED TU | catculateo |
|  |  |  |  | calculateu | ACTIVITY |
| FGIL REACTIJN |  | L.UWER | UPPER | ACTIVITIFS | (PERCEMT) |
|  |  |  |  |  |  |
| AL 27 (N,A)NAC4. | $C D$ | 0.700 E 00 | 1.000101 | 0.1295 | -87.05 |
| Allig7( $\mathrm{N}, \mathrm{G}$ ) AUL98 | CD | 4.250E-00 | 4. $500 \mathrm{E}-05$ | 3.5384 | 253.84 |
| CL59T7FGTCO60 | CD | $8.400 E=01$ | 1. 1000 -04 | 3.5733 | 257.35 |
| C(Cojs (N,G)Cun4 | CD | 3.400E-07 | 3.60.)E-01 | 3.6736 | 267.36 |
| FF5O( $\mathrm{N}, \mathrm{G}) \mathrm{FE} 59$ | $C D$ | $8.400 \mathrm{E}-07$ | 9.200E-03 | 3.9233 | 292.33 |
| FE54 (N,PJ54NN | CD. | 2.300 EO | 7.400 E 00 | 0.0897 | -91.03 |
| IM115(N,N) (N1.15M | CD | 1.000E 00 | 5.000E 00 | 0.0920 | -90.30 |
| NISU(N,P)COSO | CD | 2.000500 | 7.40 OE 00 | 0.0908 | -91.92 |
| 7k90(NJ2N)2RE9 | CD | 1.300E 07 | 1.900E O1 | 0.1512 | -83.88 |
| T140(N,P)SC46 | CD | 3,700E 00 | 1.300 E 01 | 0.0996 | -90.04 |
| T147(N, P) S $\mathrm{C}_{4} \mathrm{~T}^{7}$ | Co | 1.800E 00 | 7.400 E 00 | 0.0799 | -92.01. |
| TH232(NTFTFSPR | $C D$ | 1.400E 00 | 6.700500 | 0.0863 | -91.37 |
| U230(H,F)FSPR | Co | 1.400E 00 | $6.000 E 00$ | 0.0823 | -91.77 |
| C:163(N,A)C000 | CO. | 6,000E OO | 1.000E O1 | 0.1337 | -86.63 |
| - FESETNTPTMN5E | CD | $5.500 E O D$ | 1.600 E 01 | 0.1266 | -87.34 |
| MGZ4(N,P)NA24. | co | 6,700E CO | 1.600E 01 | 0.1299 | -87.01 |



InTERIM RESULTS AFTER 1 ITERATIDNS




| SULUT:JT SFELTK TM DETAMED AFTER 2 TTERETIDTS |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | GESDLUTE | TGLES | NORMALTEE TO | . OOE-IC MEV |  |
| EnERGY | OIFFEREMTIAL | INTEGRAL | DIFFERENTIAL | INTECFAL | AVERAGE ENERGY |
| (MEV) | Flux | FLUX | FIUX | FLTX | (1EEV) |
| 1.000E-10 | 2.032 E 19 | 5.597E 14 | 3.631E 04 | 1.000 E 00 | 1.478E-01 |
| 1.000E-09 | 1.93DE 20 | 5.597 E 14 | 3.503505 | 1.000500 | 1.47EE-01 |
| 1.000E-O日 | $4.393 E 20$ | 5.5792 .14 | 7.648505 | 9.96BE-01 | 1.423E-01 |
| 2.300E-08 | $4.6+4520$ | 5.522 E 14 | 8.290505 | 9.860E-01 | 1.498E-01 |
| 5.000E-05 | $2.505=20$ | $5.397 E 14$ |  | 9.5425-01 | 1.533E-01 |
| 7.000E-08 | $2.729 E 20$ | $5.306 E 14$ | 4.876505 | 9.479E~01 | 2.559E-01 |
| 1.150E-07 | 1.949E 20 | $5.199 E 14$ | $3.482 E 05$ | 9.289E-01 | 1.5915-01 |
| 1.TUE-07 | 1.280E 20 | $2.095 E 14$ | 2.287505 | 9.097t-01 | 1. $5<6 E-01$ |
| 2.550E=07 | 0.375519 | $4.983 E 14$ | $1.496 E 05$ | $8.903 E-01$ | $1.600 E-01$ |
| 3.800E-07 | 5.588: 19 | $4.674 E 14$ | 9.983E 04 | 8.716E-01 | 1.690E-01 |
| 5.500E-07 | 3.6567 I | $4.764 \mathrm{El4}$ | $6.555 E$ त4 | $8.546 \mathrm{~F}=01$ | $1.729 \mathrm{E}-01$ |
| 8.430E-07 | 2.372519 | 4.677514 | $4.238 E 04$ | 8.356F-01 | 2.7t9E-01 |
| 1.275E-00 | 1.551E 19 | $4.574 \mathrm{E}^{14}$ | 2.825504 | 8.172F-01 | 1.8ces-01 |
| 1.905E-06 | 1.029 E 19 | 4.475 E 14 | $1.839 E 04$ | 7.995E-01 | 1.848E-01 |
| 2.003E-06 | 7.002 E 18 | 4.383 E 14 | 1.251E 04 | 7.830E-01 | 1.897E-01 |
| 4.250E-06 | 4.760E 18 | 4.281814 | 8,504E 03 | 7.648E-01 | 1.932E-01 |
| 6.200E-00 | $3.296 E 18$ | C.183E 14 | 5.889503 | 7.474E-0: | 1.97TE-01 |
| 9.260E-06 | 2.279E 18 | 4.088 E 14 | $4.071 E 03$ | 7.303E-01 | 2.024E-01 |
| 1.350e-05 | 1.634818 | 3.990 E 14 | 2.901503 | $7.128 E-01$ | $2.073 \mathrm{E}-01$ |
| $2.160 \mathrm{E}-05$ | -.078 = 18 | $3.868 E 14$ | 1.925E 03 | 6.5712-01 | $2.738 \mathrm{E}-01$ |
| 3.000e-05 | $7.265 E 17$ | 3.771514 | $1.298 E 03$ | $6.737 E-01$ | 2.193E-01 |
| 4.50UE-05 | 4.541517 | $3.662 E 14$ | $8.203 E 02$ | 6.543E-01 | 2.259E-01 |
| $6.700 E-05$ | $3.077 E 17$ | 3.552516 | 5.498502 | $6.340 \mathrm{E}-01$ | $2.367 E-01$ |
| 1.000E-04 | 2,199E 17 | $3.457 E 14$ | $3.911 E 02$. | $6.175 E-01$ | 2.353E-01 |
| 1. $3=0$ - 0 - ${ }^{\text {a }}$ | i. 729517 | 3.380514 | 3.080E 02 | $6.0395-01$ | 2.447E-01 |
| 1.7025-64. | 1.3 '5E :7 | 3.359714 | 2.420502 | 2.730E-01 | 2,45CE-U1 |
| 2.200E-04. | 1.114817. | $3.251 E 14$ | 1.990802 | 3.808E-01 | 2.544E-01 |
| $2.800=-04^{\circ}$ | $8.773 E 16$ | $3.184 \mathrm{E}^{14}$ | 1.557502 | 5.688E-01 | 2.598F-01 |
| $3.6005-04$ | $6.675 E 16$ | 3.114E 14 | 1.193EJ2 | 5.3635-0! | 2.050E-0: |
| $4.5005=04$ | 5.153E 16 | $3.054 E 14$ | $0.206 E 01$ | 5.456E-01 | 2.709E-01 |
| 5.750E-04 | 3.770510 | 2.989 E 14. | b.736E 01 | 5,340E-0! | 3,767E-01 |
| 7.600 ECS | $2.376 E 16$ | 2.917514 | 5.138 E OT | $5.2105-01$ | 2.8S3E-OT |
| $9.6 \cup 0 E-04$ | 2.196816 | $2.852 E 14$ | 3.923 E 01 | 5.113E-01 | 2.8E9E-01 |
| 1.275F-03 | 2.6716 16 | $2.753 E 14$ | $3.021 E 01$ | $4.989=-01$ | 2.961E-01 |
| ! 5000 -03 | 1.3515 | $2.736{ }^{1} 14$ | ?.414E 0. | $4.8925-01$ | $3.020 E-01$ |
| 2.000t-c3 | 1.039510 | 2.t34E 14 | 1.856 E 0 ! | 4.79 E-0! | 3.080E-01 |
| $2.7005=03$ | $7.945 \pm 15$ | 2:611E 14 | 1:4?6E 0 ! | $4.655 \mathrm{E}-01$ | 3.165E-01 |

ESSLLITE VALIES TOR: ALILEDTO $1.00 E-10$ MEV

| --20-0- |  |  |  |  | AVFTFITE |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Ei:ERGr (MEV) | $\begin{gathered} \text { OIFFEKENTIAL } \\ \text { FLUX } \end{gathered}$ | INTEGRAL FLUX | $\begin{gathered} \text { DIFFERENTIAL } \\ \text { FLUX } \end{gathered}$ | integral Fll'x | ENERGY (MEV) |
|  |  |  |  |  |  |
| 3.4COE-03 | $0.233 E 15$ | 2.55jE 14 | $1.114 E 01$ | 4.505E-01 | 3.234E-01 |
| 4.500E-03 | 4.935E 15 | 2.487E 14 | R.817E 00 | 4.442E-01 | 3.322E-01 |
| 5.505:03 | 3.825 E 15 | $2.437 E .14$ | 7.02500 | $4.354 \mathrm{E}-61$ | 3.38EE-01 |
| 7.200E-03. | 3.030515 | 2.370E 14 | 5.425E 00 | $4.235 E-01$ | 3.432E-01 |
| 9.200E-03 | 2.36015 | 2.310E 14 | 4.252E 00 | 4.12TE-01 | 3.571E-01 |
| -.200E-02 | 1.704515 | 2.243E14 | 3.4马\%E 00 | $4.008 \mathrm{C}=01$ | 3.074E-01 |
| 1.503E-0, | $1.587 E 15$ | $2.106 E 14$ | 2.835E 00 | 3.906E-01. | 3.767E-01 |
| 1.900E-:2 | 1.264E 15 | 2.123E 14 | $2.259 E 00$ | $3.792 \mathrm{E}-01$ | 3.874E-01 |
| 2.350E=02 | 1.052525 | 2.040E 14 | 1.843500 | $3.645 F-01$ | $4.0215-01$ |
| 3.CJOE-02 | $8.651 E 14$ | $1.973 E 14$ | 1.546E 00 | 3.525E-01 | $4.148 \mathrm{E}-01$ |
| 4.000E-02 | 7.100E 14 | $1.904 \mathrm{E}_{14}$ | 1.208E 00 | 3.402E-01 | 4.286Ė-01 |
| 5.250E-52 | $5.8 \div J E 14$ | 1.ETSE 14 | T.043E 00 | 3.243t-01 | $4.413 \mathrm{E}=0$ ! |
| 6.0.00E-02 | $4.742 E 14$ | $1.736 E 14$ | 8.471E-01 | 3.102E-01 | 4.649E-01 |
| 6.800E-02 | 3.800514 | $1.632 E 14$ | 0.908E-01 | 2.916E-01 | 4.897E-01 |
| 1.100E-01 | 3.517614 | 1.347 ET 14. | $6.283 E=01$ | 2.764E-01 | 5.172E-01 |
| 1.350E-01 | 3.471514 | $2.459 E 14$ | 6.202E-01 | 2.007E-02 | 5.346E-01 |
| 1.600E-01 | $3.430 E 14$ | 1.372E 14 | 6.128E-01 | 2.452 EDOL | 5.591E-01 |
| 1.90JE-01 | 3.352514 | $1.270{ }^{14}$ | $6.0015-01$ | 2.268E-01 | 3.9 CEEOL |
| 2.200E-01: | 3.358E 14 | 1.168514 | 6.000E-01 | 2.086E-01 | 6.2385-01. |
| 2.550E-01 | 3.311E 14 | 1.050114 | 5.915E-01 | 1.976E-01 | 6.670E-01 |
| 2.900E-01 | 3.J52E IK | $9.343 E 13$ | 5.E32E-01 | 1.6E9E-01. | $7.160=-01$ |
| 3.200E-01. | $2.908 \mathrm{E}: 4$ | $8.398 E 13$ | 5.196E-01 | - $\times 1.500 t-01$ | 7.622E-01 |
| 3.600E-01 | 2.673 E 14 | ?.234E 13 | 4.776E-01 | 1.292E-01 | 8.301E-01 |
| 4.000 COL | 2.458 E 14 | $6.165 E 13$ | $4.372 E-01$ | 1.101E-01 | प.08zE-O1 |
| $4,5008=01$ | 2.152E 14 | 4.936E 13 | 3.845E-01 | 8.818E-02 | 1.029E OD |
| 5.200E-0.1 | $1.065 E 14$ | $3.860{ }^{\text {c }}$ i 3 | 3.011E-01 | 6.896E-02 | 1.183E 00 |
| 5.590 -01 | $1.185 E 14$ | 3.017E 13 | 2.117E-01 | 5.391E-02 | 1.367507 |
| 6.000E-01 | 7.955513 | 2.425513 | 1.421E-01 | 4.332E-02 | 1.560 E 00 |
| $6.600 \mathrm{E}-01$ | 5.2:2E 13 | $1.947 E 13$ | 9.419E-02 | -3.479E-02 | 1.7885 00 |
| 7.200E-01 | 3.218813 | 1.031513 | 5.749E-02 | 2.914E-02 | 2.001500 |
| 7.803E-01 | 201-0E 13 | 1.438 E 13 | 3.834E-02 | 2.569E-02 | $2.169 E 00$ |
| 8.400E-01 | 1.345E 13 | 1.307E 13 | $2.403 \mathrm{E}-02$ | 2:335E-02 | $2.302 E 00$ |
| 9.200E-01 | द.631E 12 | T.COLE 13 | 1.72IE-02 | 2.147E-02 | 2.430500 |
| 1.000E 00 | $7.524 E 12$ | $1.125 E 13$ | 1.344E-02 | 2.009E-02 | 2.530 E 00 |
| 1.200 E 00 | 0.136812 | 9.742 E 12 | 1.090E-02 | 1.741E-02 | 2.751E 00 |
| 1.400500000 | 5.103 L [2 | 3.515 E 12 | 7.120E-03 | 1.5टIE-02 | 2.961500 |
| 1.600E 00 | $4.324 E 12$ | 7.494E 12 | 7.725E-03 | $1.329 \hat{E}-02$ | 3.160500 |



XIII。 APPENDIX C






AVERAGE TOTAL FLUX (ABOVE 1.OOE-10 MEV) 2.064E 15

INTERJ：1 RESULTS AFTER 2 ITERATIONS

| ＇ |  | $5.00$ $A C T$ | INAL ERCENT VITY | RATIO | DEVIATTDN <br> DF MEASURED <br> FROM |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | CIMIT | TMEVT | MEASURED TU | CaLculated |
|  |  |  |  | CALCULATrd | ACTIVITY |
| FOIL．REACTION |  | LOWER | UPPER | ACTIVITIES | （PERCENT） |
|  |  |  |  |  |  |
| ALC7（N．A）NA24 | CD | 6．000E 00 | 1.600101 | 1.1442 | 14.42 |
| A1197（N，G）aU198 | CD | 4.250 － 0 O | 4．500E－05 | 1.0685 | 0.85 |
|  | CD | 1． $900 \mathrm{E}-06$ | 1．700E＝04 | 1.0272 | 2.72 |
| CUO3（N，G）CUO4 | CD | 8．400E－0．7 | 3．200E－01 | 0.8559 | －14．41 |
| FFSO（N，G）FES9 | $C D$. | 1．27）上ー00 | 1．350E－01 | 1.0 .026 | 6.26 |
| FE54（N；P）54MN | CD | 2.600500 | 3．2CnE 00 | 0.7930 | －20．64 |
| IN115（N．N）IN115M | CD | 5．000E－01 | 6.000 DO | 0.6780 | －30．20 |
| NIつ日（N， N$) \mathrm{CO58}$ | CD | 2.000 E 00 | 8．2OnE 00 | 0.9087 | －9．13 |
| TVgOtN；2NTZR89 | CO | 1．300EOT | 2．90nEOI | 1.0690 | 0.96 |
| T140（N，P）SC46 | $C D$ | $4.100 E .00$ | 1.300 E 01 | 0.8183 | －18．17 |
| T147（N，P）SC47 | CD | 1．800F 00 | 8.200 E 00 | 0.9544 | －4．5t |
| TTZE（N；PT5世48 | C0 | $6.000 E 00^{-}$ | $1.700 E O T$ | 1.5242 | 52.42 |
| THC32（N，F）FSPR | Cu | 1.400 E OO | $8.200 E 00$ | 1.0537 | 5.37 |
| Uaj日（ $N, F$ ）FSPR． | CO | 13．400E－01 | 7.400 E 00 | 1.0170 | ． 1.70 |
| CTO3TN－A）CDOO | CL | 5．000E 00 | 1.5 OOE OI | 1.0311 | 3.11 |
| FFSO（NっP）MNSO | CO | 5.500 E （1） | 1.500 E 01 | 0.9378 | －6．22 |
| MGC4（ $\mathrm{N}, \mathrm{H}$ ）ivact | $C D$ | 6. DOOE OO | 1.600 El | 1.0354 | 3.54 |




[^8]




THE FULLUWIHG FGIL MEASUREMENT ShDULD GE OLSCAR[ED BFCAUSE OF EMCESSIVE DEVIATICN GAFTEK TABULATIUN OF. THE SULUIION SUECTRUM, A NEH SOLUTIDN WILL BE CALGULATEO UMITTING. THIS FOILI

## NUMRER OF

OISCARDED FDIL




INTERIM kESUlts AFTER o lteratidns

INTERIM RESULTS AFTER I ITERATIONS

|  |  | $\begin{gathered} \text { NON } \\ .00 \\ A C T \end{gathered}$ | ERCENT <br> VIT | RATIO | $\begin{aligned} & \text { DEVIETIDIJ } \\ & \text { DF MEASURED } \\ & \text { FROM } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | LIMITS | TEEVT | MEASUREDTD | calculateu |
|  |  |  |  | CALCULATED | $\triangle C$ TVIIY |
| - fudl keaction |  | 1 IWWER | UPPER | ACTIVITIES | (PERCEHT) |
| ALC7( $\mathrm{N}, \mathrm{A}$ ) IVAC4 | CD | $6.000 E 00$ | 1.060e 01 | 1.3308 | 33.08 |
| AU197(H:G)AU198 | co | 4.25CE-C6 | 4.500E-05 | 1.3546 | 34.46 |
| C059 (N,G)CN60 | $C D$ | 1.275E-06 | 1.700E-04 | 1.3338 | 33.38 |
| CU03( $1 \mathrm{l}, \mathrm{G}) \mathrm{Cl,04}$ | CO | 1.275E-00 | $4.000 \mathrm{E}-01$ | 0.8089 | -19.11 |
| FFb8(N, G)FEら9 | $C D$ | 8.400E-07 | 2.550E-01 | 1.2239 | 22.39 |
|  | CD | $2.300 E 00$ | 7.400E 00 | 0.7062 | -29.38 |
| INILS(N,N)IN115M | CD | 5.000E-01 | 5.500E 00 | 0.4402 | -55.98 |
| NIS8(N,P)COSB | $C D$ | 1.800E 00 | 7.400E 00 | $0.7 E 42$ | -21.58 |
| ZR'90(N, 2 N$)$ ZR8? | CD | 1.300 ECO | 1.50NEOI | 1.3428 | 34.28 |
| TI46( $\mathrm{N}, \mathrm{P}) \mathrm{SC} 46$ | CD | $4.100 E O O$ | 1.200E 01 | 0.8276 | -17.24 |
| TI47(N, H ) SC47 | CD | 1.600E 00 | $7.400 E 00$ | 0.7974 | -20.26 |
| TH232(N,FTFSPR | CO | 1.400E 00 | 7.400 EOO | 0.8294 | -17.06 |
| U238(N,F)FSPR | co | $7.200 E-01$ | 6,700E OO | 0.7074 | -23.26 |
| CU63 ( $\mathrm{N}, \mathrm{A}$ ) CUt,0 | C0 | 0.000600 | 1.500 E 01 | 1.1746 | 17.46 |
| FE56TN, P) TiN56 | CD | 5.50 NE 00 | 1.500E O1 | 1.0411 | 4.11 |
| MG24(N,P)NA24 | CD | $6.000 E 00$ | 1.600E 01 | 1.1973 | 19.73 |
|  |  |  |  |  |  |
| STANDARD DEVYATITN OF MEASUREDACTIVITIES (PERCENT) 28.89 |  |  |  |  |  |

INTERIM RESULTS AFTER 2 ITERATIDNS

| NOMINAL |  |  |  |  | $\begin{aligned} & \text { DEVIATIUN } \\ & \text { OF MEASURED } \\ & \text { FROM } \end{aligned}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5.00 PERCENT |  |  |  |  |  |  |
|  |  |  |  | RATIO |  |  |
| Fuli kEACTION |  | LTMIT | TMEV! | MEASURFOTTU | calculated |  |
|  |  |  |  | $\begin{aligned} & \text { CALCUIATFD } \\ & \text { ACTIVITIFS } \\ & \hline \end{aligned}$ |  |  |
|  |  | LIIER | UPPER |  | aCTIVITY <br> (FERCENT) |  |
| AL 27 (N,A) NA 24 | Cu |  |  |  | ---0--- |  |
|  |  | 0.000 ED | l.6DOE O1 | 1.2091 | $20.91$ |  |
| A1J197(N:C)Alli98 | CD | 4.250t-06 | 4.500E-05 | 1.0730 |  |  |
| CO59(N,G)C060 | CD | 1.900E-C6 | 1.750E-04 | 1.0314 | $\begin{array}{r} 3.14 \\ -14.09 \end{array}$ |  |
| Cut3( $\mathrm{N}, \mathrm{G}$ ) CU64 | CD | 8.400t-C7 | 3.200E-01 | 0.8591 |  |  |
| FES8(N,G)FES9 | Co | 1.275E-06 | 1.350E-01 | 1.0668 | $\begin{array}{r} -14.09 \\ 6.08 \\ \hline \end{array}$ |  |
| FE54 (T, P154M/d | CD | $2.53050 \%$ | 8.2nNE 00 | 0.8390 | -16.10 |  |
| IN115(n)N)IN115M | $\begin{aligned} & C O \\ & C O \end{aligned}$ | 5.000r-01 | O.VODE OO | 0.6930 | -30.70 |  |
| NI58(N,P)CO58 |  | $2.000-00$ | 8.200 E 00 | 0.9512 | -4.88 |  |
| 2以 90 (N, $2 N$ ) 2 RB 9 | Co$C 0$ | 1.300501 | T. 900 E OI | 1.0703 | 7.03 |  |
| TI40 ( $\mathrm{N}, \mathrm{P}) \mathrm{SC} 40$ |  | 4.100 E 00 | 1.3 noe 01 | 0.8971 | -10.29 |  |
| T147( $\mathrm{N}, \mathrm{P}$ ) $5 \mathrm{C4} 7$ | $C 0$ $C O$ | 1.8ROE DO | 8.200 E 00 | 0.9843 | -1.57 |  |
| TH23C(N,FTFSPR | CO | 1.400000 | $7.400 E 00$ | 1.0610 | 6.10 |  |
| UZ3D(N,F)FSPK |  | $8.400 \mathrm{E}-01$ | 7.4 One OO | 1.0174 |  |  |
| CU63 ( $N, A$ ) Cnl 6 | Co | 6.000000 | 1.500 E 01 | 1.1144 | 1.7411.44 |  |
| Ft:56 ( $N$, P) Miv 56 |  | 5.500 E 00 | 1.501 | 1.0220 | 2.20$-\quad 11.09$ |  |
| MGZ4(N,P)NA24 | cos | 6.000e OO l.0ive O1 |  | 1.1109 |  |  |
| - STANOARD DEVIATIUN OF MEASU? ED ACTIVITIES (PERCENT) 12.72. |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |



$$
\text { STANOARD DEVTATION DF MEASURED ACTIVITIES (PERCENT) } 5.69
$$

:


| KESTLUTE MRIOES |  |  | Ma｜MAI2ED TOT．OCF－IC PEV |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| EHERGY | DIFFERENTIAL | INTESREL | DIFFERENTIAL | INTESPAL | EIIERGY |
| THFV： | F［TJX | FLUX | FLUX | FLT： | TMEVT |
| 1．000t－10 | 3．643E 19 | 1．414E 15 | $2.562 E 04$ | 1.00 UE 00 | $1.659 \mathrm{E}-01$ |
| 1．000E－ 50 | 3.497 V \％ | T．414E－15 | で．472E 05 | 1．000F 00 | 1．659E－O1 |
| 1．000E－1） | $3.864 E 20$ | 1.411 E | 2.732 E 05 | 9．978E－01 | 1．66．2F－01 |
| 2．300E－UA | 2.639 t 20 | 1.406 F 15 | 1．366E 05 | 9．942F－01 | 1． 6 c，8F－01 |
| 5．0nOE＝03 | $2.010 E^{2} 0$ | I．399E－15 | T：421En5 | 9．392E－01 | 1．677F－OI |
| 7．010）-68 | 1．3月7E 20 | 1．294F 15 | 1．122E 05 | $9.855 \mathrm{E}-01$ | $1.043 \mathrm{E}-01$ |
| $1.15 \cup E-07$ | $1.148 \mathrm{E} \quad 20$ | 1．3日UE 15 | $8.119 E 04$ | 9．9115－01 | $1.691 E-01$ |
| 1．70Nf－07 | 7.751 ETS | T． 3 HTET 5 | 5．474E 04 | 9．785F－05 | 1．698E－01 |
| 2．55nt－07 | $5.207 E 19$ | 1．375E 15 | 7． 0.6 2E 04 | 9．720F－01 | 1．707E－01 |
| $3.800 E-07$ | 3.574 E 19 | $1.238 \mathrm{E} \quad 15$ | $2.527 E 04$ | $9.674 \mathrm{~F}-01$ | $1.715 \mathrm{~F}-01$ |
| 5．505E－07 | 2.424 E 19 | T．352E15 | $1.714 \mathrm{E} 0^{4}$ | 9．631F－01 | 1．722E－01 |
| 8．400F－07 | 1．6．7t 19 | 1.355 E 15 | 1．151E 04 | 9．581F－01 | $1.731 \mathrm{E-01}$ |
| 1．275t－06 | 1.210 E 19 | 1．348E 15 | A．445E 03 | 9．531E－01 | 1．740E－01 |
| 1．9607－06 | 1.656719 | T． 340 E 15 | 1.147 E － 4 | $9.478 \mathrm{E}-61$ | $1.750 \mathrm{E}-01$ |
| 2．800E－06 | 1.113 F 19 | 1.326 E 15 | 7．80日E 03 | 9．374E－01 | $1.769 \mathrm{E}-01$ |
| 4．250f－06 | 7.590 F 18 | 1.310 E 15 | $5.367 E \cap 3$ | 9．2605－01 | 1．791E－01 |
| 6．30．JE－115 | 5.324 EIO | T．794ET5 | $3.765 \mathrm{E}^{03}$ | 9．150E－01 | 1.87 JE－01 |
| 9．200E－06 | 3.752 E 18 | 1．279F 15 | 2.653 E 03 | 9．041E－01 | $1.835 \mathrm{E}=01$ |
| 1．350F－05 | 1.054 El 18 | 1.263 E 15 | $7.450 E$ C2 | 8．927E－01 | 1．A58E－01 |
| $\cdots 100 E=05$ | E．b34ET7 | T．755E 15 | 6.105 E 02. | 8：S7IF－6I | 1．870E－DI |
| 1．000E－05 | 6.796 t 17 | $1.247 \mathrm{E} \quad 15$ | 4.798 E 0 ？ | 8．816E－01 | 1．881E－01 |
| $4.500 E-05$ | 0.462 E 17 | $1.237 E 15$ | $4.569 E 02$ | $8.744 \mathrm{E}-01$ | 1．997E－01 |
| $6.900 \mathrm{~F}=15$ | 4.417 F 7 | 1．22IE 15 | 3.123 E 02 | $8.635 \mathrm{E}-01$ | 1．921E－01 |
| 1．000E－04 | $3.464 t 17$ | 1．207E 15 | 2．449E 62 | 8．538E－01 | $1.943 \mathrm{E}-01$ |
| 1．35．）-1.14 | 2.650 E 17 | $1.195 E 15$ | 1．874E 02 | $8.4525-01$ | $1.902 \mathrm{E}-01$ |
| 1．70TF－ 15 | $2.069 E 17$ | T．186E 15 | $1.463 E D 2$ | 8． 3 96E－01 | I． $578 \mathrm{E}=01$ |
| 2．200t－04 | 1.431 E 17 | $1.176 E^{15}$ | 1．012E 0？ | 0．313F－01 | 1．995E－01 |
| 2．800f－0． 14 | 1．005E 17 | 1.167 E 15 | $7.105 E$ OI | 8．253F－01 | 2．010E－01 |
| $3.6001-144^{-}$ | 5．777E 16 | I．I59E－15 | 4.085 E O1 | $8.196 \mathrm{~F}=0 \mathrm{I}$ | 2．074E－01 |
| $4.500 \mathrm{E}-014$ | $4.245 E 16$ | 1.154 E 15 | 3.002 E 01 | 8．159E－01 | 2．033E－01 |
| $5.750 \mathrm{E}-\mathrm{CH}$ | 1.751 F 16 | $1.149 \mathrm{E} \quad 15$ | 1.23 HE OL | 8．172t－01 | 2．042E－01 |
| 7．600！－014 | 1.474 E 16 | T．145E－15 | 1．057E OI | 8．099E－C1 | 2．048F＝01 |
| 9．600E－0．4 | 1.317 E 16 | $1.142 \mathrm{E} \quad 15$ | 9.309 EO | 8．077E－01 | 2．053E－01 |
| 1．275E－03 | 1.358 E 10 | $1.138 \mathrm{E} \quad 15$ | $\bigcirc .599 E D 0$ | 8．048E－01 | 2．0nlt－01 |
| 1．609F＝03 | 1．177E 16 | 1．134E－15 | B． $391 \mathrm{E}^{-10}$ | 8．717F－01 | 2．06：9F－01 |
| $2.000 E=03$ | 1.068 F 16 | 1.129 E 15 | 7．549E On | 7．9H3F－01 | 2．¢77E－01 |
| 2．700F－03 | $8.839 \mathrm{E} \quad 15$ | 1.122 E 15 | $6.250 E C O$ | $7.931 \mathrm{~F}-01$ | 2．0¢1E－01 |



XIV. ARTBMDIX D

Results of llonte Cerlo error anelysis for $R-3$
ahalysis ge integral fiuence

| ENERG： | true | MINJS | MEAN | PLUS |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1．0J00E－10 | 4．9E47E＋14 | $4.5095 E+14$ | $5.0307 E+14$ | 5．4549E＋14 | $9.3 E$ |
| 1．ここのJE－09． | 4．9547E＋14 | 4.5 C55E＋14 | $5.0307 E+14$ | $5.4549 E+14$ | 9.38 |
| 1．OCODE－CE | $4.9 E-7 E+14$ | $4.5095 E+14$ | $5.0307 E+14$ ． | 5．4549E＋14 | 9.38 |
| 2．3000E－CE | 4．9E－7E＋14 | $4.50955+14$ | $5.0307 E+14$ | $5.4545 E+14$ | 5.38 |
| 5．SOCNE－OE | 4．9E47E＋14 | 4.5 ．95E＋14 | $5.0307 E+14$ | 5．4545E＋14 | 9.38 |
| 7．COOSE－08 | 4．9E－7E＋ 14 | $4.5095 E+14$ | $5.0307 E+14$ | 5．4549E＋14 | 9.38 |
| 1．：500E－07 | $4.9847 E+14$ | $4.5995 E+14$ | 5．0307E＋14 | 5．45455＋14 | 5.38 |
| 1．700うE－07 | $4.9843 E+14$ | $4.5086 E+14$ | $5.0298 E+14$ | $5.4540 E+14$ | 9.36 |
| 2．55．0E－07 | $4.9522 E+14$ | 4．5j45E＋14 | $5.0259 E+14$ | $5.4503 E+14$ | 9.39 |
| 3．6002F－C7 | $4.9751 E+14$ | 4．4F31E＋14 | $5.0145 E+14$ | 5．4398E＋14 | 9.42 |
| 5．500～E－07 | $4.9619 E+14$ | $4.46795+14$ | $4.9902 E+14$ | $5.41{ }^{7} 5 \underline{5}+14$ | 9．50 |
| 8．40005－07 | $4.92205+14$ | $4.3954 E+14$ | $4.9256 E+14$ | 5．3598E＋14 | 9.77 |
|  | $4.84=95+14$ | 4．2202E＋14 | 4．8こ50E＋14 | 5．2447E＋14 | 10.57 |
| 1．900cF－0E | $4.730: 5+14$ | $4.0315 E+14$ | $4.6591 E-14$ | 5．1317E＋14 | 11.65 |
| 二． 2 aOne－0e | $4.03 t 5 E+14$ | 3．89205＋14 | 4．5059E\＄14 | $5.0236 E+14$ | 12．13 |
| $4.25005-06$ | $4.5279 E+14$ | $3.7814 E+14$ | $4.4044 E+14$ | $4.91895+14$ | ：2．46 |
| t．30005－06 | $4.4234 E+14$ | 3．7052i +14 | 4．3656E＋： 4 | $4.8327 E+14$ | 12.77 |
| S．iJRoE－06 | $4.3: 695+14$ | 3．5515E＋14 | $4.2574 E+14$ | $4.7195 E+14$ | 13.44 |
| 1．2505E－C5 | 4．cist $E+14$ | $3.40765+14$ | $4.143 C E+14$ | $4.4174 E+16$ | 14．3： |
| C． $20005-05$ | $4.075 E E+14$ | E．2205E＋14 | 4．0058E＋14 | 4．500こE＋14 | 15．58 |
| 3．009JE－05 | 3．574．E＋14 | 3．1231E＋14 | こ．90t7E＋14 | $4.60745+i 4$ | 16.24 |
| 4．50C）－05 | 3．8tJEE＋14 | 2．97らヒE＋24 | 3．7972E＋14 | 4.2995 －： 4 | 16.78 |
| 6．¢0ces－cs | $3.7427 E+14$ | 2．8750E＋14 | 3．6®39E＋14 | $4.18235+14$ | 17.13 |
| －． 1000 O－C4 | $3.04235+14$ | C． $8335 E+14$ | 3．5BEうE＋14 | $4.58515+14$ | 17.59 |
| j．2500E－04 | 3．505 $25+14$ | 2．7331E＊：4 | 3．5j12E＋14 | $4 . \operatorname{COSCE+14}$ | 17．75 |
| 1．700：-64 | 3．4590E＋14 | $2.67765+14$ | $3.4501 E+14$ | 3．0．395E＋14 | －7．53 |
| 2． 2 COOE－U4 | $3.4268 E+14$ | 2．6114E＋14 | $3.3740 E+14$ | $3.8540 E+14$ | 16.65 |
| 2．EOUOE－O4 | $3.3573 E+14$ | C．5471E＋14 | $3.2947 E+14$ | 3．7630E＋i4 | 18.09 |
| 3．60005－C4 | 3．2E50E＋14 | 2．4836E＋14 | $3.2195 E+14$ | $3.6804 E+14$ | 18．22 |
| 4．50．00E－04 | $3.2233 E+14$ | C．4302E＋1L | 3．1592E＋14 | $3.6154 E+14$ | 18．39 |
| 5．75こうE－04 | 3．15？－5＋14 | 2．3765F＋12 | $3.0962 E+14$ | 3． $5455 \mathrm{E}+14$ | 1E．59 |
| $7.60095-04$ | 2．0E4 5 S +14 | 2．3207E＊14 | 3．0̃こうEかi4 | J．40̇1EE－14 | 1E．S． |
| O．0．00． $0-04$ | 5．0650E＋14 | 2．2713E＋j4 | 2．7552E＋14 | 3．3847E＋14 | 16.46 |
| 1．275＾：－5j | $2.9563 E+14$ | 2．2114E＋14 | 2．8769E＋14 | 3．29tSE＋14 | 12．47 |
| 1．00こJF－03 | 2．ts7iE＋14 | 2．16585＋14 | 2．F．93E－14 | 3．2312E＋14 | 1E．52 |
| －．OUJE－02 | 2．84J3E＋14 | $2.12175+14$ | 2．7644E＋14 | 3．1624E＋14 | 16．56 |
| C．7009E－C2 | 2．76 $205+14$ | 二．0345E＋14 |  | 3．：74 5 E +14 | 1E．tE |
| 3．40C3E－C3 | 2．7C5DE +14 | $2.9895 E+14$ | 2．t347E +14 | 3．0109E＋14 | ！E，t？ |
| 4．こ0．0nE－03 | $2.6324 E+14$ | $1.9359 E+14$ | 2．5600E＋14 | 2． $5334 E+14$ | 1E． 73 |
| 5．5002E－03 | 2．5802E＋14 | $1.8981 E+14$ | 2．5175E＋14 | 2．E78EE＋14 | 98．77 |
| － $2002=-03$ | 2． 5 CZ9E +14 | $1.846 E 5+14$ | C．4492E＋14 | 2．FC15E＋14 | 18．78 |
|  | 2．4ヶ̇．E－14 | －． $80 . C 3 E+14$ | 2． $208 \mathrm{EPE}+14$ | $2.73375+14$ | 1E．E3 |
| 1．200つE－02 | 2．37： $5=4.4$ | $1.7489 E+14$ | 2．32こ5E＋14 | 2．1．601E +14 | 18， 51 |
| 1．500nF－02 | 2．3133E＋14 | $1.67605+i 6$ | 2． 2655 E＋14 | 2．59CEE－14 | 15．03 |
| 1．500った－C2 | $2.2403 E+14$ | $1.6706 E+14$ | 2．2045E＋14 | 2．5248E＋14 | 19.23 |
| C．5502F－02 | 2．1579E＋14 | $1.5002 E+14$ | 2．2220E＋14 | 2．4378E＋14 | 25，53 |
| 3．2000E－02 | $2.0863 E+14$ | 1．5315E－14 | 2．0547E＋14 | 2．375EE＋14 | 19．81 |




##  Nu onnow 










Results of lonte Carlo analysis for V－1

ANALYSIS OF ILTEGRAL F：UENCE

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| ．030je－08 |
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| －6コ） |
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| ． 200 |
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| 2.60 |
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| ． 75 |
| 7.60 |
| C． 6 |
| ． 27 |
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| 4.50 |
| ． 500 |
| ． $20395-0$ |
| 4.200 |
| 200 |
| 50 |
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| ．0882E＋15 |
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| ． $05025+15$ |
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| ．08E？E＋15 |
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| ． $0502 \mathrm{E}+15$ |
| －880E＋15 |
| ．0875こ＋15 |
|  |
| ．0¢3 |
| －0775E＋15 |
| ． $0072 \mathrm{E}+15$ |
| － |
| 0392 |
| ．0243 |
| ．0151E＋15 |
| 257：E＋15 |
| ．98？ |
| ． 96 ごム + |
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| ，155E＋ |
| ． 47 |
| ． 3494 E． 14 |
| ． 250 |
| ．1338 |
| ． $03 ? 1$ |
| ． 95 ？ 3 |
| 93＇， |
| 80.99 |
| ． $80: 3 E$ |
| ． $70325+14$ |
| ． 5963 E＋14 |
| ． 5350 E＋14 |
| 8，45335－14 |
| ． 4 － 2 OE＋14 |
| ．327E5＋14 |
| 5．2：$=5-14$ |
| 1 |
| 33st＋14 |
| 7．94： 6 （E＋14 |
| ．8275E＋ |
| ．7379E＋ |
| ．6297E＋14 |
|  |
| 2794 E |


| MIN：JS | MEAN |
| :---: | :---: |
| $8.6813 \mathrm{E}+14$ | 1．2：54E＋15 |
| 8．6313E＋14 | 1．C：54E＋15 |
| $8.6813 \pm+14$ | 1．0：54E＋15 |
| 8．6813E＋14 | 1．0154E＋15 |
| $6.6813 \mathrm{E}+14$ | 1．C154E＋15 |
| E．6813E＋14 | $1.0154 E+15$ |
| E．69i2E＋14 | $1.0154 E+15$ |
| t． $6302 \mathrm{E}+14$ | 1．0153E＋！5 |
| E．6705E＋14 | 2．C150E＋15 |
| 8．6665E＋14 | 1．0142E＋15 |
| 8.6456 E－ 14 | 1．01245＋15 |
| $8.5+37 E+14$ | 1．0．976E＋15 |
| $8.4453 E+14$ | 9．9925E＋14 |
| 5．3262E－14 | $9.98935+14$ |
| 8．1749E＋14 | $9.74545+14$ |
| ．9395E＋24 | $9.5666 E+14$ |
| $7.8302 E+54$ | $9.4: 00 E+14$ |
| 7．7312E－14 | $9.3155 E+14$ |
| $7.6435 E+14$ | $9.2333 E+14$ |
| $7.5502 E+14$ | 9．1404E＋14 |
| $7.4529 E+14$ | $9.0010 E+14$ |
| $7.3440 E+14$ | $8.9428 E+14$ |
| $7.1765 E+14$ | 8．776 2 5＋14 |
| $7.0328 E+14$ | $8.6374 E+14$ |
| $0.95435+14$ | 6．5142E＋24 |
| 6.805 － $5+14$ | $8.4186 E+14$ |
| $6.7028=+14$ | 8．3122E＋14 |
| $6.6213 E+14$ | 8． $2049 \mathrm{E}+14$ |
| 6．5505E＋14 | $6.1312 E+14$ |
| t．52こ．E．14 | 2．39335 + ！ 4 |
| ．4？03E＋：4 | $8.0566 E+14$ |
| $0.4334 E+14$ | 7．9902E＋14 |
| 6．3996E＋14 | $7.8567 E+14$ |
| 6．1702E－1\％ | $7.7184 E+14$ |
| $6.1180 \mathrm{E}+: 4$ | 7．05395＊${ }^{\text {c }}$ |
| 6．0745E＊p | －．604．E－14 |
| 6．0．359E＋14 | 7．5200E＋14 |
| 5．9．967E＋14 | 7．6572E＋14 |
| $5.99645+14$ | 7．3507E＋14 |
| 5．8S20E－14 | $7.3052 \mathrm{E}+14$ |
| $5.7767 E+14$ | ？．2017E＋14 |
| 5．7132E＋14 | $7.1234 E+14$ |
| $5.6342 E+14$ | 7． $\mathrm{C} 358 \mathrm{E}+14$ |
| 5．570．E +14 | 6．9402E＋14 |
| $5.4908 E+14$ | 6．9522E＋14 |
| 5，3510E＋14 | t．6947E＋14 |
| $5.2361 E+16$ | $6.5431 E+14$ |


| $\begin{aligned} & \text { PLUS } \\ & \text { Lo? } 3 \text { IEE } 15 \end{aligned}$ | 17.78 |
| :---: | :---: |
| 1． $232 \mathrm{BE}+15$ | 17.79 |
| 1．231EE＋15 | 17．78 |
| 1．2316E4 15 | 17.76 |
| 1． $231 E E+15$ | 17.75 |
| 1．2318E＋15 | 17.78 |
| 1．c319E＋15 | 17.79 |
| $1.2317 E+15$ | 17.79 |
| 1．2315E＋15 | 17.79 |
| $1.2300 E+15$ | 17.81 |
| 1．2289E＋15 | 17.86 |
| 1．2163E＋i5 | 17.98 |
| 1．2079E＋25 | 18.20 |
| 1．：976E＋：5 | 18.46 |
| 1．1635E＋15 | 18．79 |
| 1．1662E＋15 | 19.20 |
| $1 .: 509 E+15$ | 19.50 |
| 1．1413E＋15 | 19.77 |
| 1．1329E＋15 | 19.96 |
| 1．1242E＋15 | 2 C .17 |
| 1．：$: 53 \mathrm{E}+15$ | 20．35 |
| 1．：035E＋15 | 20.64 |
| $1.0674 E+15$ | 21.07 |
| $1.2741 E+15$ | 21.47 |
| $1.0702 E+15$ | 21.82 |
| $1.0601 E+15$ | 22.03 |
| $1.0473 \mathrm{E}+15$ | 22.13 |
| 1． $0.255 E+15$ | 22.13 |
| 1．0161E＋15 | 22.15 |
| 1．0116E＋15 | 22.19 |
| 1．C076E＋15 | 22.24 |
| $1.0000 E+15$ | 22.35 |
| 9．882dE＋14 | 22.74 |
| 9．7556E＋14 | 23.20 |
| 9．6929Ed 14 | 23.36 |
| $9.6423 E+14$ | 23．49 |
| 9．5332E＋14 | 23.65 |
| 9．4933E＋14 | 23．8！ |
| $9.7679 E+14$ | 23.90 |
|  | 24.08 |
| 9．2043E＋j4 | 34.17 |
| 9．1910E＋14 | 24.23 |
| 9． $1728 \mathrm{E}+14$ | 24．29 |
| 2．9910E＋14 | 24.37 |
| 8．8701E＋14 | 24.49 |
| B．6806E＋14 | 24.62 |
| B． $4.952 \mathrm{E}+14$ | 24.73 |




##   












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[^1]:    * For more details see section IV.

[^2]:    * The code used in this woris is modified so that the energy intervals are reduced from 620 (original SAlFD-II) to 100. See section VI.

[^3]:    * The burn-up case will be considered separately in this section.

[^4]:    * This reactor $\because: a s$ decomissioned in jenuary, 1978

[^5]:    * -inis mpozrem vas devejoped at ALRR.

[^6]:    * From M. S. Wechsler's internal report: Radioactivity of thorium in ALRR, dated 7-22-1976, page 1 (flux determined by flux integral method).

[^7]:    * Private communication from Larry Greenwood of Argonne TationEj Leboratory.

[^8]:    

